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# Rock Engineering: *Background*

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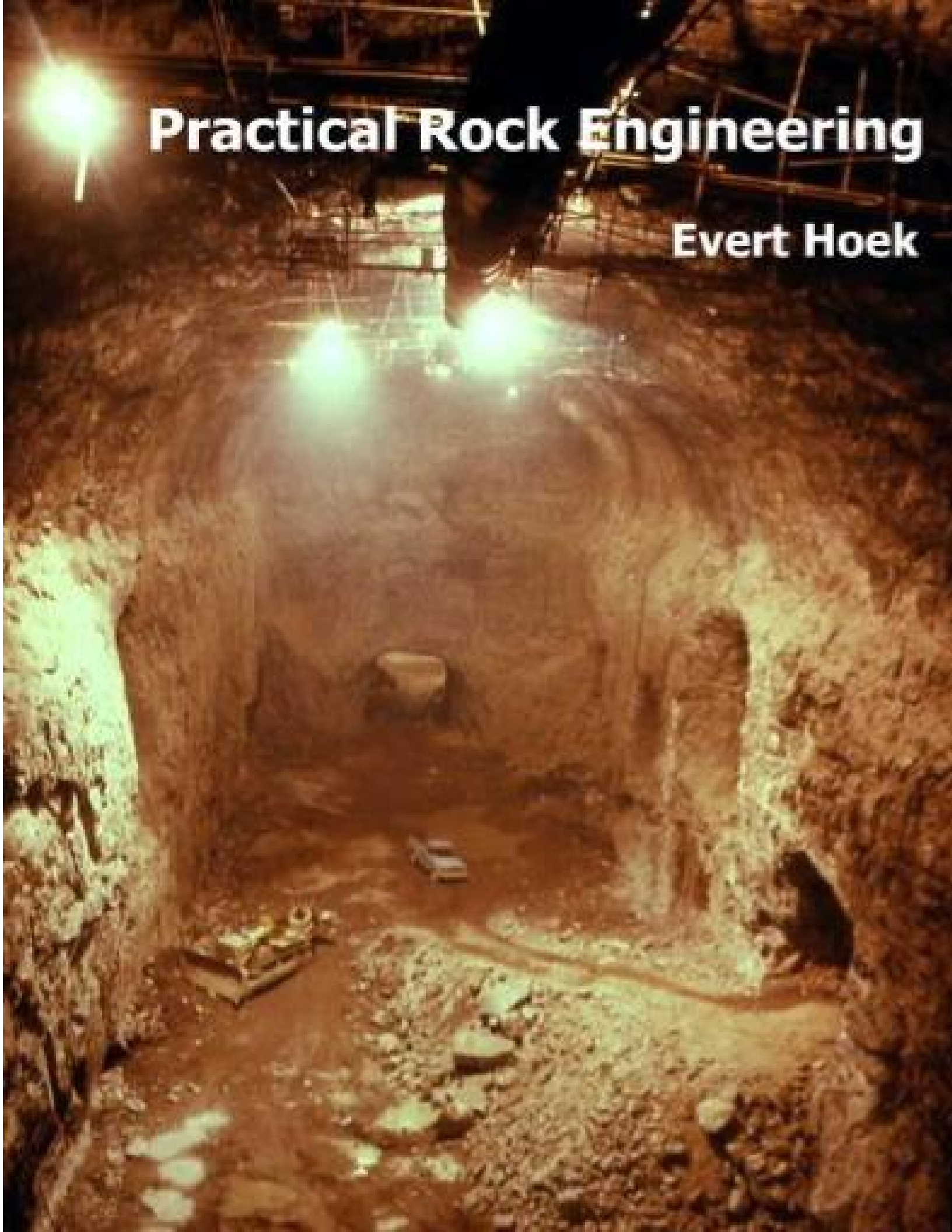
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# Practical Rock Engineering

Evert Hoek



# Preface

These notes were originally prepared during the period 1987 to 1993 for undergraduate and graduate courses in rock engineering at the University of Toronto. While some revisions were made in 2000 these were difficult because the notes had been formatted as a book with sequential chapter and page numbering. Any changes required reformatting the entire set of notes and this made it impractical to carry out regular updates.

In 2006 it was decided that a major revision was required in order to incorporate significant developments in rock engineering during the 20 years since the notes were originally written. The existing document was broken into a series of completely self-contained chapters, each with its own page numbering and references. This means that individual chapters can be updated at any time and that new chapters can be inserted as required.

The notes are intended to provide an insight into practical rock engineering to students, geotechnical engineers and engineering geologists. Case histories are used, wherever possible, to illustrate the methods currently used by practicing engineers. No attempt has been made to include recent research findings which have not yet found their way into everyday practical application. These research findings are adequately covered in conference proceedings, journals and on the Internet.

It is emphasised that these are notes are not a formal text. They have not been and will not be published in their present form and the contents will be revised from time to time to meet the needs of particular audiences.

Readers are encouraged to send their comments, corrections, criticisms and suggestions to me at the address given below. These contributions will help me to improve the notes for the future.



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# The development of rock engineering

## Introduction

We tend to think of rock engineering as a modern discipline and yet, as early as 1773, Coulomb included results of tests on rocks from Bordeaux in a paper read before the French Academy in Paris (Coulomb, 1776, Heyman, 1972). French engineers started construction of the Panama Canal in 1884 and this task was taken over by the US Army Corps of Engineers in 1908. In the half century between 1910 and 1964, 60 slides were recorded in cuts along the canal and, although these slides were not analysed in rock mechanics terms, recent work by the US Corps of Engineers (Lutton et al, 1979) shows that these slides were predominantly controlled by structural discontinuities and that modern rock mechanics concepts are fully applicable to the analysis of these failures. In discussing the Panama Canal slides in his Presidential Address to the first international conference on Soil Mechanics and Foundation Engineering in 1936, Karl Terzaghi (Terzaghi, 1936, Terzaghi and Voight, 1979) said ‘The catastrophic descent of the slopes of the deepest cut of the Panama Canal issued a warning that we were overstepping the limits of our ability to predict the consequences of our actions ....’.

In 1920 Josef Stini started teaching ‘Technical Geology’ at the Vienna Technical University and before he died in 1958 he had published 333 papers and books (Müller, 1979). He founded the journal *Geologie und Bauwesen*, the forerunner of today’s journal *Rock Mechanics*, and was probably the first to emphasise the importance of structural discontinuities on the engineering behaviour of rock masses.

Other notable scientists and engineers from a variety of disciplines did some interesting work on rock behaviour during the early part of this century. von Karman (1911), King (1912), Griggs (1936), Ide (1936), and Terzaghi (1945) all worked on the failure of rock materials. In 1921 Griffith proposed his theory of brittle material failure and, in 1931 Bucky started using a centrifuge to study the failure of mine models under simulated gravity loading.

None of these persons would have classified themselves as rock engineers or rock mechanics engineers - the title had not been invented at that time - but all of them made significant contributions to the fundamental basis of the subject as we know it today. I have made no attempt to provide an exhaustive list of papers related to rock mechanics which were published before 1960 but the references given above will show that important developments in the subject were taking place well before that date.

The early 1960s were very important in the general development of rock engineering world-wide because a number of catastrophic failures occurred which clearly demonstrated that, in rock as well as in soil, ‘we were over-stepping the limits of our ability to predict the consequences of our actions’ (Terzaghi and Voight, 1979).

## *The development of rock engineering*

In December 1959 the foundation of the Malpasset concrete arch dam in France failed and the resulting flood killed about 450 people (Figure 1). In October 1963 about 2500 people in the Italian town of Longarone were killed as a result of a landslide generated wave which overtopped the Vajont dam (Figure 2). These two disasters had a major impact on rock mechanics in civil engineering and a large number of papers were written on the possible causes of the failures (Jaeger, 1972).



Figure 1: Remains of the Malpasset Dam as seen today. Photograph by Mark Diederichs, 2003.



Figure 2a: The Vajont dam during impounding of the reservoir. In the middle distance, in the centre of the picture, is Mount Toc with the unstable slope visible as a white scar on the mountain side above the waterline.





Figure 2b: During the filling of the Vajont reservoir the toe of the slope on Mount Toc was submerged and this precipitated a slide. The mound of debris from the slide is visible in the central part of the photograph. The very rapid descent of the slide material displaced the water in the reservoir causing a 100 m high wave to overtop the dam wall. The dam itself, visible in the foreground, was largely undamaged.



Figure 2c: The town of Longarone, located downstream of the Vajont dam, before the Mount Toc failure in October 1963.





Figure 2d: The remains of the town of Longarone after the flood caused by the overtopping of the Vajont dam as a result of the Mount Toc failure. More than 2000 persons were killed in this flood.



Figure 2e: The remains of the Vajont dam perched above the present town of Longarone. Photograph by Mark Diederichs, 2003.

In 1960 a coal mine at Coalbrook in South Africa collapsed with the loss of 432 lives. This event was responsible for the initiation of an intensive research programme which resulted in major advances in the methods used for designing coal pillars (Salamon and Munro, 1967).

The formal development of rock engineering or rock mechanics, as it was originally known, as an engineering discipline in its own right dates from this period in the early 1960s and I will attempt to review these developments in the following chapters of these notes. I consider myself extremely fortunate to have been intimately involved in the subject since 1958. I have also been fortunate to have been in positions which required extensive travel and which have brought me into personal contact with most of the persons with whom the development of modern rock engineering is associated.

### **Rockbursts and elastic theory**

Rockbursts are explosive failures of rock which occur when very high stress concentrations are induced around underground openings. The problem is particularly acute in deep level mining in hard brittle rock. Figure 3 shows the damage resulting from a rockburst in an underground mine. The deep level gold mines in the Witwatersrand area in South Africa, the Kolar gold mines in India, the nickel mines centred on Sudbury in Canada, the mines in the Coeur d'Alene area in Idaho in the USA and the gold mines in the Kalgoorlie area in Australia, are amongst the mines which have suffered from rockburst problems.



Figure 3: The results of a rockburst in an underground mine in brittle rock subjected to very high stresses.



As early as 1935 the deep level nickel mines near Sudbury were experiencing rockburst problems and a report on these problems was prepared by Morrison in 1942. Morrison also worked on rockburst problems in the Kolar gold fields in India and describes some of these problems in his book, *A Philosophy of Ground Control* (1976).

Early work on rockbursts in South African gold mines was reported by Gane et al (1946) and a summary of rockburst research up to 1966 was presented by Cook et al (1966). Work on the seismic location of rockbursts by Cook (1963) resulted in a significant improvement of our understanding of the mechanics of rockbursting and laid the foundations for the microseismic monitoring systems which are now common in mines with rockburst problems.

A characteristic of almost all rockbursts is that they occur in highly stressed, brittle rock. Consequently, the analysis of stresses induced around underground mining excavations, a key in the generation of rockbursts, can be dealt with by means of the theory of elasticity. Much of the early work in rock mechanics applied to mining was focused on the problem of rockbursts and this work is dominated by theoretical solutions which assume isotropic elastic rock and which make no provision for the role of structural discontinuities. In the first edition of Jaeger and Cook's book, *Fundamentals of Rock Mechanics* (1969), mention of structural discontinuities occurs on about a dozen of the 500 pages of the book. This comment does not imply criticism of this outstanding book but it illustrates the dominance of elastic theory in the approach to rock mechanics associated with deep-level mining problems. Books by Coates (1966) and by Obert and Duvall (1967) reflect the same emphasis on elastic theory.

This emphasis on the use of elastic theory for the study of rock mechanics problems was particularly strong in the English speaking world and it had both advantages and disadvantages. The disadvantage was that it ignored the critical role of structural features. The advantage was that the tremendous concentration of effort on this approach resulted in advances which may not have occurred if the approach had been more general.

Many mines and large civil engineering projects have benefited from this early work in the application of elastic theory and most of the modern underground excavation design methods have their origins in this work.

### **Discontinuous rock masses**

Stini was one of the pioneers of rock mechanics in Europe and he emphasised the importance of structural discontinuities in controlling the behaviour of rock masses (Müller, 1979). Stini was involved in a wide range of near-surface civil engineering works and it is not surprising that his emphasis was on the role of discontinuities since this was obviously the dominant problem in all his work. Similarly, the text book by Talobre (1957), reflecting the French approach to rock mechanics, recognised the role of structure to a much greater extent than did the texts of Jaeger and Cook, Coates and Obert and Duvall.

A major impetus was given to this work by the Malpasset dam failure and the Vajont disaster mentioned earlier. The outstanding work by Londe and his co-workers in France (Londe, 1965, Londe et al, 1969, 1970) and by Wittke (1965) and John (1968) in Germany laid the foundation for the three-dimensional structural analyses which we have available today. Figure 4 shows a wedge failure controlled by two intersecting structural features in the bench of an open pit mine.



Figure 4: A wedge failure controlled by intersecting structural features in the rock mass forming the bench of an open pit mine.

## **Rock Engineering**

Civil and mining engineers have been building structures on or in rock for centuries (Figure 5) and the principles of rock engineering have been understood for a long time. Rock mechanics is merely a formal expression of some of these principles and it is only during the past few decades that the theory and practice in this subject have come together in the discipline which we know today as rock engineering. A particularly important event in the development of the subject was the merging of elastic theory, which dominated the English language literature on the subject, with the discontinuum approach of the Europeans. The gradual recognition that rock could act both as an elastic material and a discontinuous mass resulted in a much more mature approach to the subject than had previously been the case. At the same time, the subject borrowed techniques for dealing with soft rocks and clays from soil mechanics and recognised the importance of viscoelastic and rheological behaviour in materials such as salt and potash.



Figure 5: The 1036 m long Eupalinos water supply tunnel was built in 530 BC on the Greek island of Samos. This is the first known tunnel to have been built from two portals and the two drives met with a very small error.

The photograph was provided by Professor Paul Marinos of the National Technical University of Athens.



I should point out that significant work on rock mechanics was being carried out in countries such as Russia, Japan and China during the 25 years covered by this review but, due to language differences, this work was almost unknown in the English language and European rock mechanics centres and almost none of it was incorporated into the literature produced by these centres.

### **Geological data collection**

The corner-stone of any practical rock mechanics analysis is the geological model and the geological data base upon which the definition of rock types, structural discontinuities and material properties is based. Even the most sophisticated analysis can become a meaningless exercise if the geological model upon which it is based is inadequate or inaccurate.

Methods for the collection of geological data have not changed a great deal over the past 25 years and there is still no acceptable substitute for the field mapping and core logging. There have been some advances in the equipment used for such logging and a typical example is the electronic compass illustrated in Figure 6. The emergence of geological engineering or engineering geology as recognised university degree courses has been an important step in the development of rock engineering. These courses train geologists to be specialists in the recognition and interpretation of geological information which is significant in engineering design. These geological engineers, following in the tradition started by Stini in the 1920s, play an increasingly important role in modern rock engineering.



Figure 6: A Clar electronic geological compass manufactured by F.W. Breihapt in Germany.

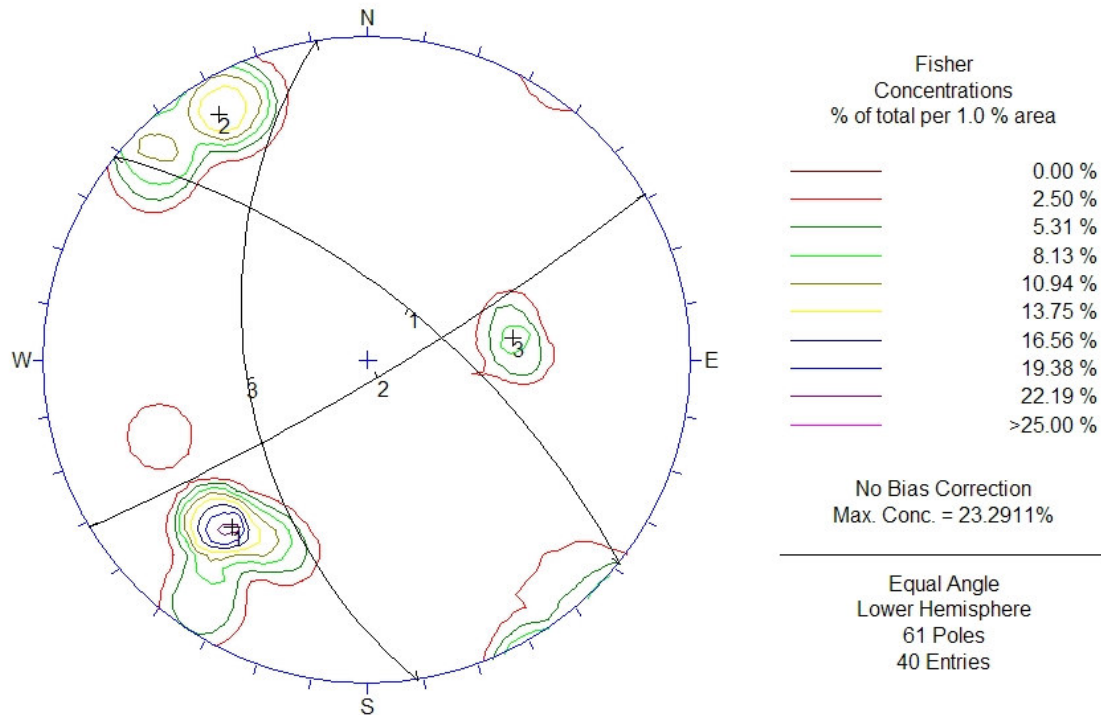


Figure 7: Plot of structural features using the program DIPS.

Once the geological data have been collected, computer processing of this data can be of considerable assistance in plotting the information and in the interpretation of statistically significant trends. Figure 7 illustrates a plot of contoured pole concentrations and corresponding great circles produced by the program DIPS developed at the University of Toronto and now available from Rocscience Inc.

Surface and down-hole geophysical tools and devices such as borehole cameras have been available for several years and their reliability and usefulness has gradually improved as electronic components and manufacturing techniques have advanced. However, current capital and operating costs of these tools are high and these factors, together with uncertainties associated with the interpretation of the information obtained from them, have tended to restrict their use in rock engineering. It is probable that the use of these tools will become more widespread in years to come as further developments occur.

### **Laboratory testing of rock**

There has always been a tendency to equate rock mechanics with laboratory testing of rock specimens and hence laboratory testing has played a disproportionately large role in the subject. This does not imply that laboratory testing is not important but I would suggest that only about 10 percent of a well balanced rock mechanics program should be allocated to laboratory testing.

Laboratory testing techniques have been borrowed from civil and mechanical engineering and have remained largely unaltered for the past 25 years. An exception has been the development of servo-controlled stiff testing machines which permit the determination of the complete stress-strain curve for rocks. This information is important in the design of underground excavations since the properties of the failed rock surrounding the excavations have a significant influence upon the stability of the excavations.

### **Rock mass classification**

A major deficiency of laboratory testing of rock specimens is that the specimens are limited in size and therefore represent a very small and highly selective sample of the rock mass from which they were removed. In a typical engineering project, the samples tested in the laboratory represent only a very small fraction of one percent of the volume of the rock mass. In addition, since only those specimens which survive the collection and preparation process are tested, the results of these tests represent a highly biased sample. How then can these results be used to estimate the properties of the in situ rock mass?

In an attempt to provide guidance on the properties of rock masses a number of rock mass classification systems have been developed. In Japan, for example, there are 7 rock mass classification systems, each one developed to meet a particular set of needs.

Probably the most widely known classifications, at least in the English speaking world, are the RMR system of Bieniawski (1973, 1974) and the Q system of Barton, Lien and Lunde (1974). The classifications include information on the strength of the intact rock material, the spacing, number and surface properties of the structural discontinuities as well as allowances for the influence of subsurface groundwater, in situ stresses and the orientation and inclination of dominant discontinuities. These classifications were developed primarily for the estimation of the support requirements in tunnels but their use has been expanded to cover many other fields.

Provided that they are used within the limits within which they were developed, as discussed by Palmstrom and Broch (2006), these rock mass classification systems can be very useful practical engineering tools, not only because they provide a starting point for the design of tunnel support but also because they force users to examine the properties of the rock mass in a very systematic manner.

### **Rock mass strength**

One of the major problems confronting designers of engineering structures in rock is that of estimating the strength of the rock mass. This rock mass is usually made up of an interlocking matrix of discrete blocks. These blocks may have been weathered or altered to varying degrees and the contact surfaces between the blocks may vary from clean and fresh to clay covered and slickensided.



Determination of the strength of an in situ rock mass by laboratory type testing is generally not practical. Hence this strength must be estimated from geological observations and from test results on individual rock pieces or rock surfaces which have been removed from the rock mass. This question has been discussed extensively by Hoek and Brown (1980) who used the results of theoretical (Hoek, 1968) and model studies (Brown, 1970, Ladanyi and Archambault, 1970) and the limited amount of available strength data, to develop an empirical failure criterion for jointed rock masses. Hoek (1983) also proposed that the rock mass classification system of Bieniawski could be used for estimating the rock mass constants required for this empirical failure criterion. This classification proved to be adequate for better quality rock masses but it soon became obvious that a new classification was required for the very weak tectonically disturbed rock masses associated with the major mountain chains of the Alps, the Himalayas and the Andes.

The Geological Strength Index (GSI) was introduced by Hoek in 1994 and this Index was subsequently modified and expanded as experience was gained on its application to practical rock engineering problems. Marinos and Hoek (2000, 2001) published the chart reproduced in Figure 8 for use in estimating the properties of heterogeneous rock masses such as flysch (Figure 9).

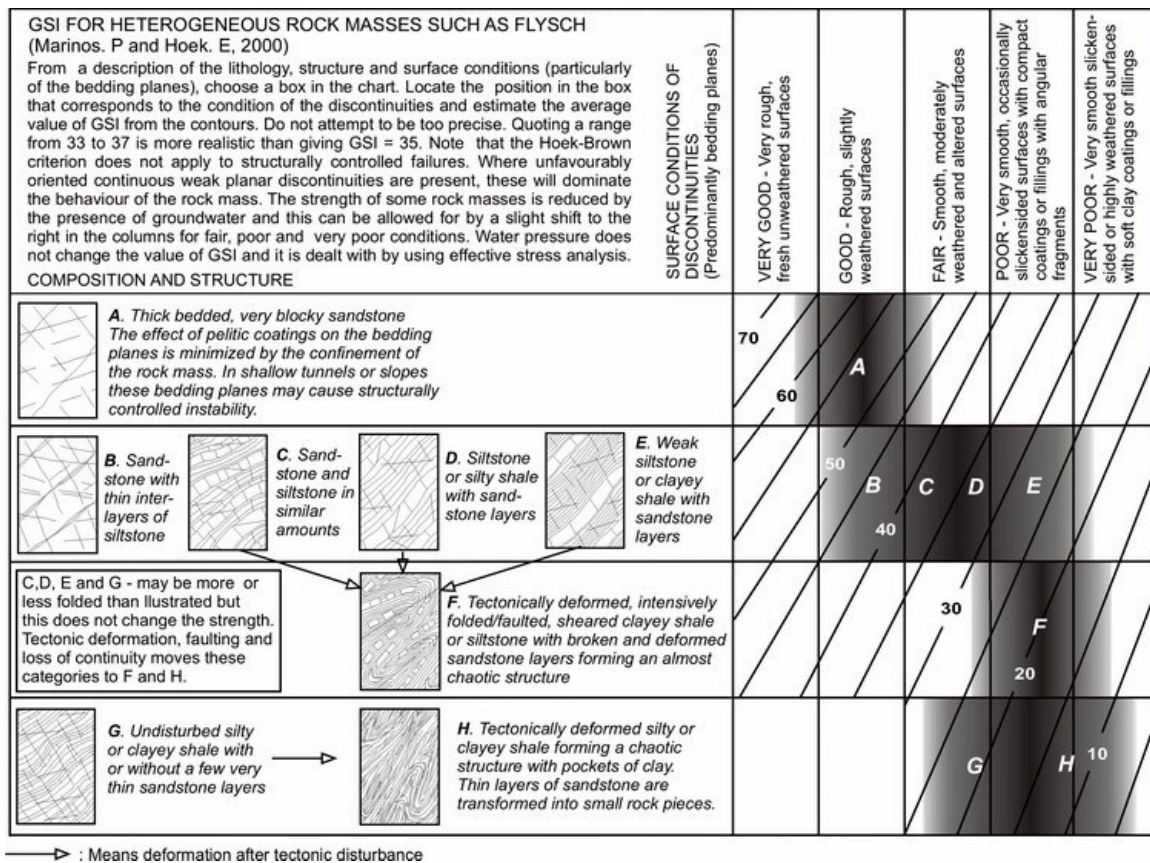


Figure 8: Geological Strength Index for heterogeneous rock masses such as flysch from Marinos and Hoek 2000.



Figure 9: Various grades of flysch in an exposure in the Pindos mountains of northern Greece.

Practical application of the GSI system and the Hoek-Brown failure criterion in a number of engineering projects around the world have shown that the system gives reasonable estimates of the strength of a wide variety of rock masses. These estimates have to be refined and adjusted for individual conditions, usually based upon back analysis of tunnel or slope behaviour, but they provide a sound basis for design analyses. The most recent version of the Hoek-Brown criterion has been published by Hoek, Carranza-Torres and Corkum (2002) and this paper, together with a program called RocLab for implementing the criterion, can be downloaded from the Internet at [www.rocscience.com](http://www.rocscience.com).

### **In situ stress measurements**

The stability of deep underground excavations depends upon the strength of the rock mass surrounding the excavations and upon the stresses induced in this rock. These induced stresses are a function of the shape of the excavations and the in situ stresses which existed before the creation of the excavations. The magnitudes of pre-existing in situ stresses have been found to vary widely, depending upon the geological history of the rock mass in which they are measured (Hoek and Brown, 1980). Theoretical predictions of these stresses are considered to be unreliable and, hence, measurement of the actual in situ stresses is necessary for major underground excavation design. A phenomenon which is frequently observed in massive rock subjected to high in situ stresses is 'core dishing', illustrated in Figure 10.





Figure 10: Disking of a 150 mm core of granite as a result of high in situ stresses.

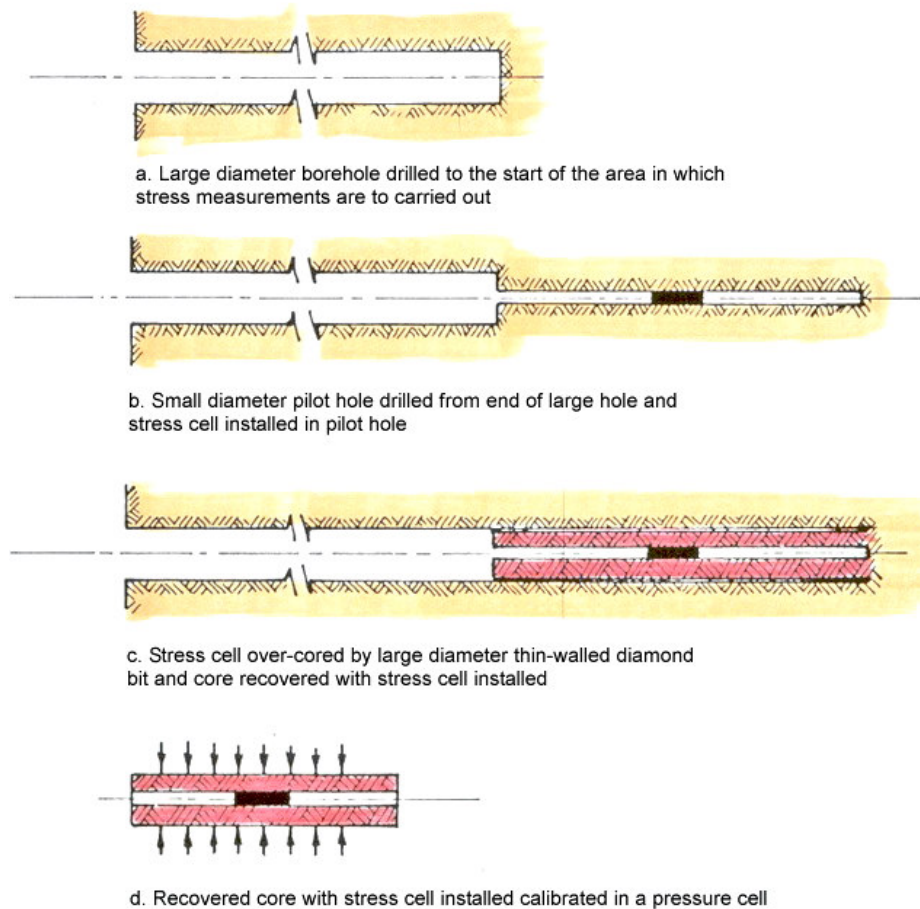


Figure 11: Typical sequence of over-coring stress measurements.





Figure 12: A cell for measuring the in situ triaxial stress field in a rock mass, developed in Australia (Worotnicki and Walton 1976). The hollow cylinder (on the left) is filled with adhesive which is extruded when the piston (on the right) is forced into the cylinder.

During early site investigations, when no underground access is available, the only practical method for measuring in situ stresses is by hydrofracturing (Haimson, 1978) in which the hydraulic pressure required to open existing cracks is used to estimate in situ stress levels. Once underground access is available, over-coring techniques for in situ stress measurement (Leeman and Hayes, 1966, Worotnicki and Walton, 1976) can be used and, provided that sufficient care is taken in executing the measurements, the results are usually adequate for design purposes. A typical over-coring sequence for in situ stress measurement is illustrated in Figure 11 and one of the instruments used for such measurement is illustrated in Figure 12.

### **Groundwater problems**

The presence of large volumes of groundwater is an operational problem in tunnelling but water pressures are generally not too serious a problem in underground excavation engineering. Exceptions are pressure tunnels associated with hydroelectric projects. In these cases, inadequate confining stresses due to insufficient depth of burial of the tunnel can cause serious problems in the tunnel and in the adjacent slopes. The steel linings for these tunnels can cost several thousand dollars per metre and are frequently a critical factor in the design of a hydroelectric project. The installation of a steel tunnel lining is illustrated in Figure 13.



Figure 13: Installation of steel lining in a pressure tunnel in a hydroelectric project.

Groundwater pressures are a major factor in all slope stability problems and an understanding of the role of subsurface groundwater is an essential requirement for any meaningful slope design (Hoek and Bray, 1981, Brown, 1982).

While the actual distributions of water pressures in rock slopes are probably much more complex than the simple distributions normally assumed in slope stability analyses (Freeze and Cherry, 1979), sensitivity studies based upon these simple assumptions are generally adequate for the design of drainage systems (Masur and Kaufman, 1962). Monitoring of groundwater pressures by means of piezometers (Brown, 1982) is the most reliable means of establishing the input parameters for these groundwater models and for checking upon the effectiveness of drainage measures.

In the case of dams, forces generated by the water acting on the upstream face of the dam and water pressures generated in the foundations are critical in the assessment of the stability of the dam. Estimates of the water pressure distribution in the foundations and of

the influence of grout and drainage curtains upon this distribution have to be made with care since they have a significant impact upon the overall dam and foundation design (Soos, 1979).

The major advances that have been made in the groundwater field during the past decades have been in the understanding of the transport of pollutants by groundwater. Because of the urgency associated with nuclear and toxic waste disposal in industrialised countries, there has been a concentration of research effort in this field and advances have been impressive. The results of this research do not have a direct impact on conventional geotechnical engineering but there have been many indirect benefits from the development of instrumentation and computer software which can be applied to both waste disposal and geotechnical problems.

### **Rock reinforcement and support design**

Safety during construction and long term stability are factors that have to be considered by the designers of excavations in rock. It is not unusual for these requirements to lead to a need for the installation of some form of rock reinforcement or support. Fortunately, practical developments in this field have been significant during the past 25 years and today's rock engineer has a wide choice of reinforcement systems and tunnel lining techniques. In particular, the development of shotcrete has made a major contribution to modern underground construction.

There has been considerable confusion in the use of the terms "reinforcement" and "support" in rock engineering and it is important for the reader to understand the different roles of these two important systems.

Rock reinforcement, as the name implies, is used to improve the strength and/or deformational behaviour of a rock mass in much the same way that steel bars are used to improve the performance of reinforced concrete. The reinforcement generally consists of bolts or cables that are placed in the rock mass in such a way that they provide confinement or restraint to counteract loosening and movement of the rock blocks. They may or may not be tensioned, depending upon the sequence of installation, and they may or may not be grouted, depending upon whether they are temporary or permanent. In general, rock reinforcement is only fully effective in reasonably frictional rock masses of moderate to high strength. Such rock masses permit effective anchoring of the reinforcement and they also develop the interlocking required to benefit from the confinement provided by the reinforcement. In reinforced rock masses, mesh and/or shotcrete play an important role in bridging the gap between adjacent bolt or anchor heads and in preventing progressive ravelling of small pieces of rock that are not confined by the reinforcement.

For weak to very weak rock masses that are more cohesive than frictional, reinforcement is less effective and, in the case of extremely weak materials, may not work at all. In these cases it is more appropriate to use support rather than reinforcement. This support, which generally consists of steel sets and shotcrete or concrete linings in different



combinations, must act as a load bearing structural shell to be fully effective in failing weak ground. The primary function of the support is to limit deformation of the rock or soil mass surrounding the tunnel and the sequence of installation, in relation to the advance of the tunnel face, is critically important. The capacity of the structural shell must be calculated on the basis of the bending moments and axial thrusts that are generated in the support elements and connections. In the case of large tunnels in very weak, highly stressed ground, where top heading and bench or multiple headings are used, temporary internal support shells may be required in order to prevent collapse of the temporary excavation boundaries. The development of shotcrete has been extremely important in weak ground tunnelling since it permits the rapid installation of a temporary or permanent load bearing lining with embedded reinforcement as required.

The use of long untensioned grouted cables in underground hard rock mining (Clifford, 1974, Fuller, 1983, Hunt and Askew, 1977, Brady and Brown, 1985) has been a particularly important innovation which has resulted in significant improvements in safety and mining costs in massive ore bodies. The lessons learned from these mining systems have been applied with considerable success in civil engineering and the use of untensioned dowels, installed as close as possible to the advancing face, has many advantages in high speed tunnel construction. The use of untensioned grouted cables or reinforcing bars has also proved to be a very effective and economical technique in rock slope stabilisation. This reinforcement is installed progressively as the slope is benched downward and it is very effective in knitting the rock mass together and preventing the initiation of raveling.

The design of both rock reinforcement and support have benefited greatly from the evolution of personal computers and the development of very powerful and user-friendly software. Whereas, in the past, these designs were based on empirical rules or classification schemes derived from experience, it is now possible to study a wide range of excavation geometries, excavation sequences, rock mass properties and reinforcement or support options by means of numerical models. This does not imply that every metre of every excavation has to be subjected to such analyses but it does mean that, once a reliable geological model has been established, the designer can choose a few reinforcement or support systems and optimize these for the typical conditions anticipated.

### **Excavation methods in rock**

As pointed out earlier, the strength of jointed rock masses is very dependent upon the interlocking between individual rock pieces. This interlocking is easily destroyed and careless blasting during excavation is one of the most common causes of underground excavation instability. The following quotation is taken from a paper by Holmberg and Persson (1980):

The innocent rock mass is often blamed for insufficient stability that is actually the result of rough and careless blasting. Where no precautions have been taken to avoid blasting damage, no knowledge of the real stability of the undisturbed rock can be gained from

looking at the remaining rock wall. What one sees are the sad remains of what could have been a perfectly safe and stable rock face.

Techniques for controlling blast damage in rock are well-known (Svanholm et al, 1977, Langefors and Kihlstrom, 1963, Hagan, 1980) but it is sometimes difficult to persuade owners and contractors that the application of these techniques is worthwhile. Experience in projects in which carefully controlled blasting has been used generally shows that the amount of reinforcement can be reduced significantly and that the overall cost of excavation and support is lower than in the case of poorly blasted excavations (Hoek, 1982). Examples of poor and good quality blasting in tunnels are illustrated in Figures 1.10 and 1.11.

Machine excavation is a technique which causes very little disturbance to the rock surrounding an underground excavation. A wide range of tunnelling machines have been developed over the past 25 years and these machines are now capable of working in almost all rock types (Robbins, 1976, McFeat-Smith, 1982). Further development of these machines can be expected and it is probable that machine excavation will play a much more important role in future tunnelling than it does today.

### **Analytical tools**

Analytical models have always played an important role in rock mechanics. The earliest models date back to closed form solutions such as that for calculating the stresses surrounding a circular hole in a stressed plate published by Kirsch in 1898. The development of the computer in the early 1960s made possible the use of iterative numerical techniques such as finite element (Clough, 1960), boundary element (Crouch and Starfield, 1983), discrete element (Cundall, 1971) and combinations of these methods (von Kimmelman et al, 1984, Lorig and Brady, 1984). These have become almost universal tools in rock mechanics.

The computer has also made it much more convenient to use powerful limit equilibrium methods (Sarma, 1979, Brown and Ferguson, 1979, Shi and Goodman, 1981, Warburton, 1981) and probabilistic approaches (McMahon, 1971, Morriss and Stoter, 1983, Priest and Brown, 1982, Read and Lye, 1983) for rock mechanics studies.

The advent of the micro-computer and the rapid developments which have taken place in inexpensive hardware have brought us to the era of a computer on every professional's desk. The power of these machines is transforming our approach to rock mechanics analysis since it is now possible to perform a large number of sensitivity or probabilistic studies in a fraction of the time which was required for a single analysis a few years ago. Given the inherently inhomogeneous nature of rock masses, such sensitivity studies enable us to explore the influence of variations in the value of each input parameter and to base our engineering judgements upon the rate of change in the calculated value rather than on a single answer.



Figure 1.10: An example of poor blasting in a tunnel.



Figure 1.11: An example of good blasting in a tunnel.



## **Conclusions**

Over the past 25 years, rock mechanics has developed into a mature subject which is built on a solid foundation of geology and engineering mechanics. Individuals drawn from many different disciplines have contributed to this subject and have developed a wide range of practical tools and techniques. There is still a great deal of room for development, innovation and improvement in almost every aspect of the subject and it is a field which will continue to provide exciting challenges for many years to come.

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# When is a rock engineering design acceptable

## Introduction

When is a design in rock engineering acceptable? The aim of the following text<sup>1</sup> is to demonstrate that there are no simple universal rules for acceptability nor are there standard factors of safety which can be used to guarantee that a rock structure will be safe and that it will perform adequately. Each design is unique and the acceptability of the structure has to be considered in terms of the particular set of circumstances, rock types, design loads and end uses for which it is intended. The responsibility of the geotechnical engineer is to find a safe and economical solution which is compatible with all the constraints which apply to the project. Such a solution should be based upon engineering judgement guided by practical and theoretical studies such as stability or deformation analyses, if and when these analyses are applicable.

Tables 1 to 4 summarise some of the typical problems, critical parameters, analysis methods and acceptability criteria which apply to a number of different rock engineering structures. These examples have been drawn from my own consulting experience and I make no claims that this is a complete list nor do I expect readers to agree with all of the items which I have included under the various headings. The purpose of presenting these tables is to demonstrate the diversity of problems and criteria which have to be considered and to emphasise the dangers of attempting to use standard factors of safety or other acceptability criteria.

In order to amplify some of the items included in Tables 1 to 4, several case histories will be discussed in terms of the factors which were considered and the acceptability criteria which were used.

## Landslides in reservoirs

The presence of unstable slopes in reservoirs is a major concern for the designers of dams for hydroelectric and irrigation projects. The Vajont failure in 1963 alerted the engineering community of the danger of underestimating the potential for the mobilisation of existing landslides as a result of submergence of the slide toe during impounding of the reservoir.

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<sup>1</sup>Based upon the text of the Müller lecture presented at the 7th Congress of the International Society for Rock Mechanics held in Aachen, Germany, in September 1991.

Table 1 : Typical problems, critical parameters, methods of analysis and acceptability criteria for slopes.

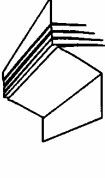
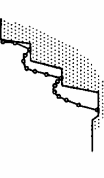
STRUCTURE	TYPICAL PROBLEMS	CRITICAL PARAMETERS	ANALYSIS METHODS	ACCEPTABILITY CRITERIA
 Landslides.	Complex failure along a circular or near circular failure surface involving sliding on faults and other structural features as well as failure of intact materials.	<ul style="list-style-type: none"> <li>• Presence of regional faults.</li> <li>• Shear strength of materials along failure surface.</li> <li>• Groundwater distribution in slope, particularly in response to rainfall or to submergence of slope toe.</li> <li>• Potential earthquake loading.</li> </ul>	Limit equilibrium methods which allow for non-circular failure surfaces can be used to estimate changes in factor of safety as a result of drainage or slope profile changes. Numerical methods such as finite element or discrete element analysis can be used to investigate failure mechanisms and history of slope displacement.	Absolute value of factor of safety has little meaning but rate of change of factor of safety can be used to judge effectiveness of remedial measures. Long term monitoring of surface and subsurface displacements in slope is the only practical means of evaluating slope behaviour and effectiveness of remedial action.
 Soil or heavily jointed rock slopes.	Circular failure along a spoon-shaped surface through soil or heavily jointed rock masses.	<ul style="list-style-type: none"> <li>• Height and angle of slope face.</li> <li>• Shear strength of materials along failure surface.</li> <li>• Groundwater distribution in slope.</li> <li>• Potential surcharge or earthquake loading.</li> </ul>	Two-dimensional limit equilibrium methods which include automatic searching for the critical failure surface are used for parametric studies of factor of safety. Probability analyses, three-dimensional limit equilibrium analyses or numerical stress analyses are occasionally used to investigate unusual slope problems.	Factor of safety $> 1.3$ for "temporary" slopes with minimal risk of damage. Factor of safety $> 1.5$ for "permanent" slopes with significant risk of damage. Where displacements are critical, numerical analyses of slope deformation may be required and higher factors of safety will generally apply in these cases.
 Jointed rock slopes.	Planar or wedge sliding on one structural feature or along the line of intersection of two structural features.	<ul style="list-style-type: none"> <li>• Slope height, angle and orientation.</li> <li>• Dip and strike of structural features.</li> <li>• Groundwater distribution in slope.</li> <li>• Potential earthquake loading.</li> <li>• Sequence of excavation and support installation.</li> </ul>	Limit equilibrium analyses which determine three-dimensional sliding modes are used for parametric studies on factor of safety. Failure probability analyses, based upon distribution of structural orientations and shear strengths, are useful for some applications.	Factor of safety $> 1.3$ for "temporary" slopes with minimal risk of damage. Factor of safety $> 1.5$ for "permanent" slopes with significant risk of damage. Probability of failure of 10 to 15% may be acceptable for open pit mine slopes where cost of clean up is less than cost of stabilization.
 Vertically jointed rock slopes.	Toppling of columns separated from the rock mass by steeply dipping structural features which are parallel or nearly parallel to the slope face.	<ul style="list-style-type: none"> <li>• Slope height, angle and orientation.</li> <li>• Dip and strike of structural features.</li> <li>• Groundwater distribution in slope.</li> <li>• Potential earthquake loading.</li> </ul>	Crude limit equilibrium analyses of simplified block models are useful for estimating potential for toppling and sliding. Discrete element models of simplified slope geometry can be used for exploring toppling failure mechanisms.	No generally acceptable criterion for toppling failure is available although potential for toppling is usually obvious. Monitoring of slope displacements is the only practical means of determining slope behaviour and effectiveness of remedial measures.
 Loose boulders on rock slopes.	Sliding, rolling, falling and bouncing of loose rocks and boulders on the slope.	<ul style="list-style-type: none"> <li>• Geometry of slope.</li> <li>• Presence of loose boulders.</li> <li>• Coefficients of restitution of materials forming slope.</li> <li>• Presence of structures to arrest falling and bouncing rocks.</li> </ul>	Calculation of trajectories of falling or bouncing rocks based upon velocity changes at each impact is generally adequate. Monte Carlo analyses of many trajectories based upon variation of slope geometry and surface properties give useful information on distribution of fallen rocks.	Location of fallen rock or distribution of a large number of fallen rocks will give an indication of the magnitude of the potential rockfall problem and of the effectiveness of remedial measures such as draped mesh, catch fences and ditches at the toe of the slope.

Table 2 : Typical problems, critical parameters, methods of analysis and acceptability criteria for dams and foundations.

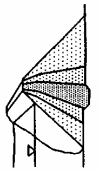
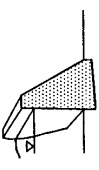
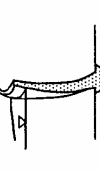
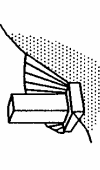
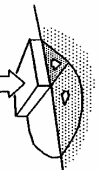
STRUCTURE	TYPICAL PROBLEMS	CRITICAL PARAMETERS	ANALYSIS METHODS	ACCEPTABILITY CRITERIA
 Zoned fill dams.	Circular or near-circular failure of dam, particularly during rapid drawdown. Foundation failure on weak seams. Piping and erosion of core.	<ul style="list-style-type: none"> <li>• Presence of weak or permeable zones in foundation.</li> <li>• Shear strength, durability, gradation and placement of dam construction materials, particularly filters.</li> <li>• Effectiveness of grout curtain and drainage system.</li> <li>• Stability of reservoir slopes.</li> </ul>	Seepage analyses are required to determine water pressure and velocity distribution through dam and abutments. Limit equilibrium methods should be used for parametric studies of stability. Numerical methods can be used to investigate dynamic response of dam during earthquakes.	Safety factor >1.5 for full pool with steady state seepage; >1.3 for end of construction with no reservoir loading and undissipated foundation porewater pressures; >1.2 for probable maximum flood with steady state seepage and >1.0 for full pool with steady state seepage and maximum credible horizontal pseudo-static seismic loading.
 Gravity dams.	Shear failure of interface between concrete and rock or of foundation rock. Tension crack formation at heel of dam. Leakage through foundation and abutments.	<ul style="list-style-type: none"> <li>• Presence of weak or permeable zones in rock mass.</li> <li>• Shear strength of interface between concrete and rock.</li> <li>• Shear strength of rock mass.</li> <li>• Effectiveness of grout curtain and drainage system.</li> <li>• Stability of reservoir slopes.</li> </ul>	Parametric studies using limit equilibrium methods should be used to investigate sliding on the interface between concrete and rock and sliding on weak seams in the foundation. A large number of trial failure surfaces are required unless a non-circular failure analysis with automatic detection of critical failure surfaces is available.	Safety factor against foundation failure should exceed 1.5 for normal full pool operating conditions provided that conservative shear strength values are used ( $c' \approx 0$ ). Safety factor > 1.3 for probable maximum flood (PMF). Safety factor > 1 for extreme loading - maximum credible earthquake and PMF.
 Arch dams.	Shear failure in foundation or abutments. Cracking of arch due to differential settlements of foundation. Leakage through foundations or abutments.	<ul style="list-style-type: none"> <li>• Presence of weak, deformable or permeable zones in rock mass.</li> <li>• Orientation, inclination and shear strength of structural features.</li> <li>• Effectiveness of grout curtain and drainage system.</li> <li>• Stability of reservoir slopes.</li> </ul>	Limit equilibrium methods are used for parametric studies of three-dimensional sliding modes in the foundation and abutments, including the influence of water pressures and reinforcement. Three-dimensional numerical analyses are required to determine stresses and displacements in the concrete arch.	Safety factor against foundation failure >1.5 for normal full pool operating conditions and >1.3 for probable maximum flood conditions provided that conservative shear strength values are used ( $c' \approx 0$ ). Stresses and deformations in concrete arch should be within allowable working levels defined in concrete specifications.
 Foundations on rock slopes.	Slope failure resulting from excessive foundation loading. Differential settlement due to anisotropic deformation properties of foundation rocks.	<ul style="list-style-type: none"> <li>• Orientation, inclination and shear strength of structural features in rock mass forming foundation.</li> <li>• Presence of inclined layers with significantly different deformation properties.</li> <li>• Groundwater distribution in slope.</li> </ul>	Limit equilibrium analyses of potential planar or wedge failures in the foundation or in adjacent slopes are used for parametric studies of factor of safety. Numerical analyses can be used to determine foundation deformation, particularly for anisotropic rock masses.	Factor of safety against sliding of any potential foundation wedges or blocks should exceed 1.5 for normal operating conditions. Differential settlement should be within limits specified by structural engineers.
 Foundations on soft rock or soil.	Bearing capacity failure resulting from shear failure of soils or weak rocks underlying foundation slab.	<ul style="list-style-type: none"> <li>• Shear strength of soil or jointed rock materials.</li> <li>• Groundwater distribution in soil or rock foundation.</li> <li>• Foundation loading conditions and potential for earthquake loading.</li> </ul>	Limit equilibrium analyses using inclined slices and non-circular failure surfaces are used for parametric studies of factor of safety. Numerical analyses may be required to determine deformations, particularly for anisotropic foundation materials.	Bearing capacity failure should not be permitted for normal loading conditions. Differential settlement should be within limits specified by structural engineers.



Table 3 : Typical problems, critical parameters, methods of analysis and acceptability criteria for underground civil engineering excavations.

STRUCTURE	TYPICAL PROBLEMS	CRITICAL PARAMETERS	ANALYSIS METHODS	ACCEPTABILITY CRITERIA
 <p>Pressure tunnels in hydro-power projects.</p>	<p>Excessive leakage from unlined or concrete lined tunnels.</p> <p>Rupture or buckling of steel lining due to rock deformation or external pressure.</p>	<ul style="list-style-type: none"> <li>Ratio of maximum hydraulic pressure in tunnel to minimum principal stress in the surrounding rock.</li> <li>Length of steel lining and effectiveness of grouting.</li> <li>Groundwater levels in the rock mass.</li> </ul>	<p>Determination of minimum cover depths along pressure tunnel route from accurate topographic maps.</p> <p>Stress analyses of sections along and across tunnel axis.</p> <p>Comparison between minimum principal stresses and maximum dynamic hydraulic pressure to determine steel lining lengths.</p>	<p>Steel lining is required where the minimum principal stress in the rock is less than 1.3 times the maximum static head for typical hydroelectric operations or 1.15 for operations with very low dynamic pressures.</p> <p>Hydraulic pressure testing in boreholes at the calculated ends of the steel lining is essential to check the design assumptions.</p>
 <p>Soft rock tunnels.</p>	<p>Rock failure where strength is exceeded by induced stresses.</p> <p>Swelling, squeezing or excessive closure if support is inadequate.</p>	<ul style="list-style-type: none"> <li>Strength of rock mass and of individual structural features.</li> <li>Swelling potential, particularly of sedimentary rocks.</li> <li>Excavation method and sequence.</li> <li>Capacity and installation sequence of support systems.</li> </ul>	<p>Stress analyses using numerical methods to determine extent of failure zones and probable displacements in the rock mass.</p> <p>Rock-support interaction analyses using closed-form or numerical methods to determine capacity and installation sequence for support and to estimate displacements in the rock mass.</p>	<p>Capacity of installed support should be sufficient to stabilize the rock mass and to limit closure to an acceptable level. Tunneling machines and internal structures must be designed for closure of the tunnel as a result of swelling or time-dependent deformation. Monitoring of deformations is an important aspect of construction control.</p>
 <p>Shallow tunnels in jointed rock.</p>	<p>Gravity driven falling or sliding wedges or blocks defined by intersecting structural features.</p> <p>Unravelling of inadequately supported surface material.</p>	<ul style="list-style-type: none"> <li>Orientation, inclination and shear strength of structural features in the rock mass.</li> <li>Shape and orientation of excavation.</li> <li>Quality of drilling and blasting during excavation.</li> <li>Capacity and installation sequence of support systems.</li> </ul>	<p>Spherical projection techniques or analytical methods are used for the determination and visualization of all potential wedges in the rock mass surrounding the tunnel.</p> <p>Limit equilibrium analyses of critical wedges are used for parametric studies on the mode of failure, factor of safety and support requirements.</p>	<p>Factor of safety, including the effects of reinforcement, should exceed 1.5 for sliding and 2.0 for falling wedges and blocks.</p> <p>Support installation sequence is critical and wedges or blocks should be identified and supported before they are fully exposed by excavation.</p> <p>Displacement monitoring is of little value.</p>
 <p>Large caverns in jointed rock.</p>	<p>Gravity driven falling or sliding wedges or tensile and shear failure of rock mass, depending upon spacing of structural features and magnitude of in situ stresses.</p>	<ul style="list-style-type: none"> <li>Shape and orientation of cavern in relation to orientation, inclination and shear strength of structural features in the rock mass.</li> <li>In situ stresses in the rock mass.</li> <li>Excavation and support sequence and quality of drilling and blasting.</li> </ul>	<p>Spherical projection techniques or analytical methods are used for the determination and visualization of all potential wedges in the rock mass. Stresses and displacements induced by each stage of cavern excavation are determined by numerical analyses and are used to estimate support requirements for the cavern roof and walls.</p>	<p>An acceptable design is achieved when numerical models indicate that the extent of failure has been controlled by installed support, that the support is not overstressed and that the displacements in the rock mass stabilize.</p> <p>Monitoring of displacements is essential to confirm design predictions.</p>
 <p>Underground nuclear waste disposal.</p>	<p>Stress and/or thermally induced spalling of the rock surrounding the excavations resulting in increased permeability and higher probability of radioactive leakage.</p>	<ul style="list-style-type: none"> <li>Orientation, inclination, permeability and shear strength of structural features in the rock mass.</li> <li>In situ and thermal stresses in the rock surrounding the excavations.</li> <li>Groundwater distribution in the rock mass.</li> </ul>	<p>Numerical analyses are used to calculate stresses and displacements induced by excavation and by thermal loading from waste canisters. Groundwater flow patterns and velocities, particularly through blast damaged zones, fissures in the rock and shaft seals are calculated using numerical methods.</p>	<p>An acceptable design requires extremely low rates of groundwater movement through the waste canister containment area in order to limit transport of radioactive material. Shafts, tunnels and canister holes must remain stable for approximately 50 years to permit retrieval of waste if necessary.</p>

Table 4 : Typical problems, critical parameters, methods of analysis and acceptability criteria for underground hard rock mining excavations.

STRUCTURE	TYPICAL PROBLEMS	CRITICAL PARAMETERS	ANALYSIS METHODS	ACCEPTABILITY CRITERIA
 <p>Pillars.</p>	Progressive spalling and slabbing of the rock mass leading to eventual pillar collapse or rockbursting.	<ul style="list-style-type: none"> <li>Strength of the rock mass forming the pillars.</li> <li>Presence of unfavourably oriented structural features.</li> <li>Pillar geometry, particularly width to height ratio.</li> <li>Overall mine geometry including extraction ratio.</li> </ul>	For horizontally bedded deposits, pillar strength from empirical relationships based upon width to height ratios and average pillar stress based on tributary area calculations are compared to give a factor of safety. For more complex mining geometry, numerical analyses including progressive pillar failure may be required.	Factor of safety for simple pillar layouts in horizontally bedded deposits should exceed 1.6 for "permanent" pillars. In cases where progressive failure of complex pillar layouts is modelled, individual pillar failures can be tolerated provided that they do not initiate "domino" failure of adjacent pillars.
 <p>Crown pillars.</p>	Caving of surface crown pillars for which the ratio of pillar depth to stope span is inadequate. Rockbursting or gradual spalling of overstressed internal crown pillars.	<ul style="list-style-type: none"> <li>Strength of the rock mass forming the pillars.</li> <li>Depth of weathering and presence of steeply dipping structural features in the case of surface crown pillars.</li> <li>In situ stress levels and geometry of internal crown pillars.</li> </ul>	Rock mass classification and limit equilibrium analyses can give useful guidance on surface crown pillar dimensions for different rock masses. Numerical analyses, including discrete element studies, can give approximate stress levels and indications of zones of potential failure.	Surface crown pillar depth to span ratio should be large enough to ensure very low probability of failure. Internal crown pillars may require extensive support to ensure stability during mining of adjacent stopes. Careful planning of mining sequence may be necessary to avoid high stress levels and rockburst problems.
 <p>Cut and fill stopes.</p>	Falls of structurally defined wedges and blocks from stope backs and hanging walls. Stress induced failures and rockbursting in high stress environments.	<ul style="list-style-type: none"> <li>Orientation, inclination and shear strength of structural features in the rock mass.</li> <li>In situ stresses in the rock mass.</li> <li>Shape and orientation of stope.</li> <li>Quality, placement and drainage of fill.</li> </ul>	Numerical analyses of stresses and displacements for each excavation stage will give some indication of potential problems. Some of the more sophisticated numerical models will permit inclusion of the support provided by fill or the reinforcement of the rock by means of grouted cables.	Local instability should be controlled by the installation of rockbolts or grouted cables to improve safety and to minimize dilution. Overall stability is controlled by the geometry and excavation sequence of the stopes and the quality and sequence of filling. Acceptable mining conditions are achieved when all the ore is recovered safely.
 <p>Non-entry stopes.</p>	Ore dilution resulting from rockfalls from stope back and walls. Rockbursting or progressive failure induced by high stresses in pillars between stopes.	<ul style="list-style-type: none"> <li>Quality and strength of the rock.</li> <li>In situ and induced stresses in the rock surrounding the excavations.</li> <li>Quality of drilling and blasting in excavation of the stope.</li> </ul>	Some empirical rules, based on rock mass classification, are available for estimating safe stope dimensions. Numerical analyses of stope layout and mining sequence, using three-dimensional analyses for complex orebody shapes, will provide indications of potential problems and estimates of support requirements.	A design of this type can be considered acceptable when safe and low cost recovery of a large proportion of the orebody has been achieved. Rockfalls in shafts and haulages are an unacceptable safety hazard and pattern support may be required. In high stress environments, local destressing may be used to reduce rockbursting.
 <p>Drawpoints and ore passes.</p>	Local rock mass failure resulting from abrasion and wear of poorly supported drawpoints or orepasses. In extreme cases this may lead to loss of stopes or orepasses.	<ul style="list-style-type: none"> <li>Quality and strength of the rock.</li> <li>In situ and induced stresses and stress changes in the rock surrounding the excavations.</li> <li>Selection and installation sequence of support.</li> </ul>	Limit equilibrium or numerical analyses are not particularly useful since the processes of wear and abrasion are not included in these models. Empirical designs based upon previous experience or trial and error methods are generally used.	The shape of the opening should be maintained for the design life of the drawpoint or orepass. Loss of control can result in serious dilution of the ore or abandonment of the excavation. Wear resistant flexible reinforcement such as grouted cables, installed during excavation of the opening, may be successful in controlling instability.

## *When is a rock engineering design acceptable*

During the construction of the Mica and Revelstoke dams on the Columbia River in British Columbia, Canada, several potential slides were investigated. Two of these, the Downie Slide, a 1.4 billion cubic metre ancient rock slide, and Dutchman's Ridge, a 115 million cubic metre potential rock slide, were given special attention because of the serious consequences which could have resulted from failure of these slides (Imrie, 1983, Lewis and Moore, 1989, Imrie, Moore and Eneqren, 1992).

The Downie Slide and Dutchman's Ridge are located in steep, narrow, V-shaped sections of the Columbia River valley which has been subjected to several episodes of glaciation. The bedrock at these sites consists mainly of Pre-Cambrian para-gneisses and schists within or on the fringe of the Shuswap Metamorphic Complex. In both cases, the potential slide planes, determined by diamond drilling and slope displacement monitoring, are relatively flat-lying outward-dipping tectonic faults or shears which daylight in the base of the river valley.

Based on thorough investigation and monitoring programs, British Columbia Hydro and Power Authority (BC Hydro) decided that remedial measures had to be taken to improve the stability of both the Downie Slide and Dutchman's Ridge. These remedial measures consisted of drainage adits extending within and/or behind the failure surfaces and supplemented by drainholes drilled from chambers excavated along the adits. Work on the Downie Slide was carried out in the period 1977 to 1982 (which included a 3 year observation period) and work on Dutchman's Ridge was carried out from 1986 to 1988.

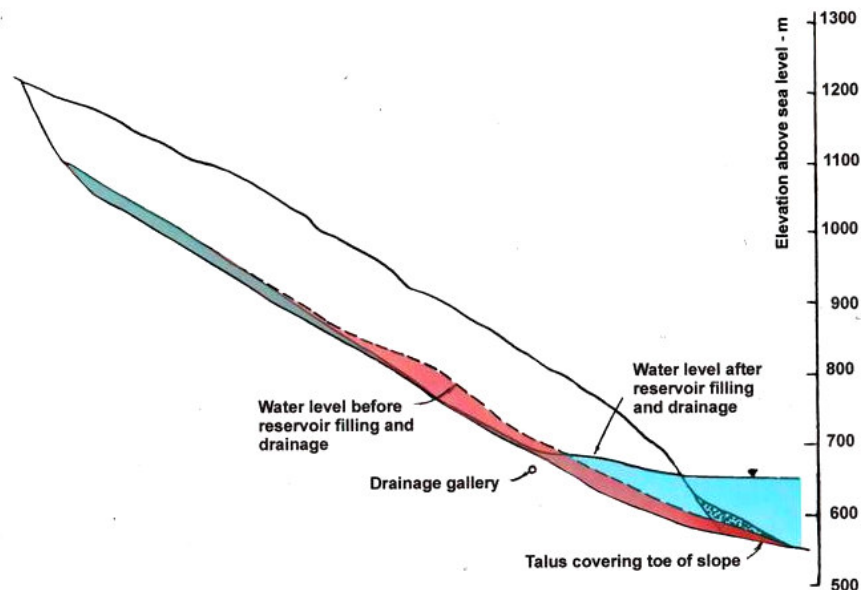


Figure 1: Section through Dutchman's Ridge showing potential slide surface and water levels before and after drainage.



## *When is a rock engineering design acceptable*

A section through Dutchman's Ridge is given in Figure 1 and this shows the water levels in the slope before reservoir filling and after reservoir filling and the construction of the drainage system. Figure 2 shows contours of reduction in water levels as a result of the installation of the drainage system which consisted of 872 m of adit and 12,000 m of drainhole drilling. Note that the drawdown area on the right hand side of the potential slide was achieved by long boreholes from the end of the drainage adit branch.

Comparative studies of the stability of the slope section shown in Figure 1, based upon a factor of safety of 1.00 for the slope after reservoir filling but before implementation of the drainage system, gave a factor of safety of 1.06 for the drained slope. This 6% improvement in factor of safety may not seem very significant to the designer of small scale rock and soil slopes but it was considered acceptable in this case for a number of reasons:

1. The factor of safety of 1.00 calculated for the undrained slope is based upon a 'back-analysis' of observed slope behaviour. Provided that the same method of analysis and shear strength parameters are used for the stability analysis of the same slope with different groundwater conditions, the ratio of the factors of safety is a very reliable indicator of the change in slope stability, even if the absolute values of the factor of safety are not accurate. Consequently, the degree of uncertainty, which has to be allowed for in slope designs where no back-analyses have been performed, can be eliminated and a lower factor of safety accepted.

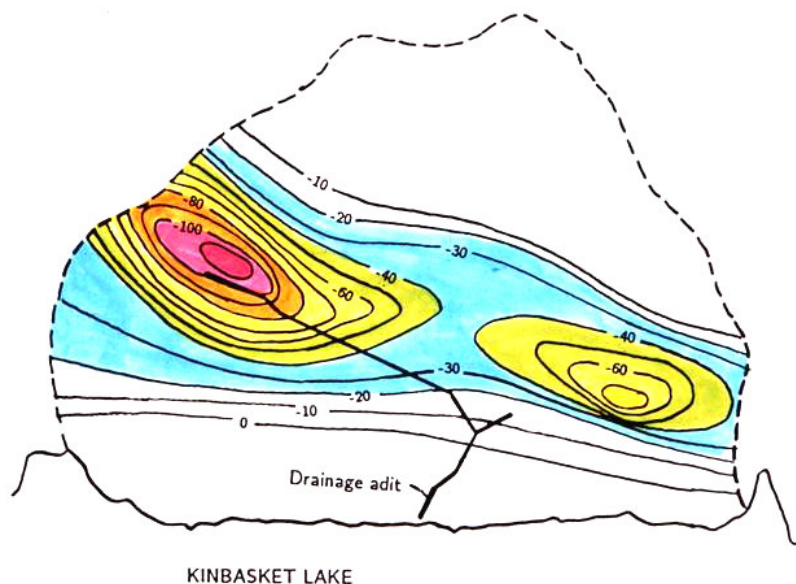


Figure 2: Contours of water level reduction (in metres) as a result of the implementation of drainage in Dutchman's Ridge.

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2. The groundwater levels in the slope were reduced by drainage to lower than the pre-reservoir conditions and the stability of the slope is at least as good if not better than these pre-reservoir conditions. This particular slope is considered to have withstood several significant earthquakes during the 10,000 years since the last episode of glaciation which is responsible for the present valley shape.
3. Possibly the most significant indicator of an improvement in stability, for both the Downie Slide and Dutchman's Ridge, has been a significant reduction in the rate of down-slope movement which has been monitored for the past 25 years. In the case of the Downie Slide, this movement has practically ceased. At Dutchman's Ridge, the movements are significantly slower and it is anticipated that they will stabilize when the drainage system has been in operation for a few more years.

## **Deformation of rock slopes**

In a slope in which the rock is jointed but where there are no significant discontinuities dipping out of the slope which could cause sliding, deformation and failure of the slope is controlled by a complex process of block rotation, tilting and sliding. In an extreme case, where the rock mass consists of near vertical joints separating columns of massive rock, toppling movement and failure may occur.

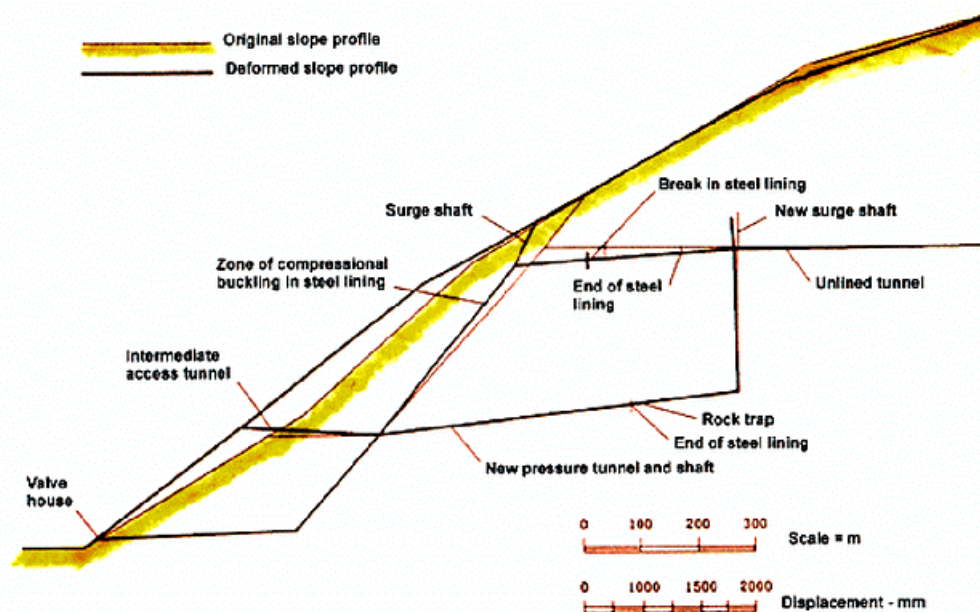


Figure 3: Cross-section through a section of the Wahleach power tunnel showing the original tunnel alignment and the location of the replacement conduit. The dashed line is the approximate location of a gradational boundary between loosened, fractured and weathered rock and more intact rock. Down-slope movement currently being monitored is well above this boundary.

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Figure 3 is a section through part of the power tunnel for the Wahleach hydroelectric project in British Columbia, Canada. A break in the steel lining in this power tunnel occurred in January 1989 and it is thought this break was caused by a slow down-slope gravitational movement caused by block rotations within a near-surface zone of loosened jointed rock.

The Wahleach project is located 120 km east of Vancouver and power is generated from 620 m of head between Wahleach Lake and a surface powerhouse located adjacent to the Fraser River. Water flows through a 3500 m long three metre diameter unlined upper tunnel, a rock trap, a 600 m two metre diameter concrete encased steel lined shaft inclined at  $48^\circ$  to the horizontal, a 300 m long lower tunnel and a 485 m long surface penstock to the powerhouse.

The tunnels were excavated mainly in granodiorite which varies from highly fractured and moderately weathered in the upper portions of the slope to moderately fractured and fresh in both the lower portions of the slope and below the highly fractured mass. Two main joint sets occur in the rock mass, one set striking parallel to the slope and the other perpendicular to it. Both dip very steeply. Average joint spacings range from 0.5 to 1 m. A few joints occur sub-parallel to the ground surface and these joints are most well developed in the ground surface adjacent to the inclined shaft. Thorough investigations failed to reveal any significant shear zones or faults conducive to sliding.

The toe of the slope is buried beneath colluvial and fan deposits from two creeks which have incised the Fraser Valley slope to form the prominence in which the inclined shaft was excavated. This prominence is crossed by several linear troughs which trend along the ground surface contours and are evidence of previous down-slope movement of the prominence. Mature trees growing in these troughs indicate a history of movement of at least several hundred years (Moore, Imrie and Baker, 1991).

The water conduit operated without incident between the initial filling in 1952 and May 1981 when leakage was first noted from the upper access adit located near the intersection of the inclined shaft and the upper tunnel (see Figure 3). This leakage stopped when two drain pipes embedded in the concrete backfill beneath the steel lining were plugged at their upstream ends. Large holes had been eroded in these drainage pipes where they were not encased in concrete and it was concluded that this corrosion was responsible for the leakage. This conclusion appeared to be valid until 25 January, 1989 when a much larger water flow occurred.

Investigations in the dewatered tunnel revealed a 150 mm wide circumferential tension crack in the steel lining of the upper tunnel, about 55 m from its intersection with the inclined shaft. In addition, eight compressional buckle zones were found in the upper portion of the inclined shaft. Subsequent investigations revealed that approximately 20 million cubic metres of rock are involved in down-slope creep which, during 1989-90, amounted to several centimetres per year and which appears to be ongoing. This down-

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slope creep appears to be related to a process of block rotation rather than to any deep seated sliding as was the case at both the Downie Slide and Dutchman's Ridge.

While discrete element models may give some indication of the overall mechanics of this type of slope deformation, there is no way in which a factor of safety, equivalent to that for sliding failure, can be calculated. Consequently, in deciding upon the remedial measures to be implemented, other factors have to be taken into consideration.

After thorough study by the BC Hydro and their consultants, it was decided to construct a replacement conduit consisting of an unlined shaft and tunnel section and a steel lined section where the rock cover is insufficient to contain the internal pressure in the tunnel. This replacement conduit, illustrated in Figure 3, will remove the steel lined portions of the system from zones in which large displacements are likely to occur in the future. This in turn will minimise the risk of a rupture of the steel lining which would inject high pressure water into the slope. It was agreed that such high pressure water leakage could be a cause for instability of the overall slope. Further studies are being undertaken to determine whether additional drainage is required in order to provide further safeguards.

Careful measurements of the displacements in the inclined shaft, the length of the steel lining cans as compared with the original specified lengths and the opening of the tensile crack in the upper portion of the steel lined tunnel, provided an overall picture of the displacements in the rock mass. These observed displacements were compared with displacement patterns computed by means of a number of numerical studies using both continuum and discrete element models and the results of these studies were used in deciding upon the location of the replacement conduit.

In addition to the construction of this replacement conduit to re-route the water away from the upper and potentially unstable part of the slope, a comprehensive displacement and water pressure monitoring system has been installed and is being monitored by BC Hydro (Baker, 1991, Tatchell, 1991).

### **Structural failures in rock masses**

In slopes, foundations and shallow underground excavations in hard rock, failure is frequently controlled by the presence of discontinuities such as faults, shear zones, bedding planes and joints. The intersection of these structural features can release blocks or wedges which can fall or slide from the surface of the excavation. Failure of the intact rock is seldom a problem in these cases where deformation and failure are caused by sliding along individual discontinuity surfaces or along lines of intersection of surfaces. Separation of planes and rotation of blocks and wedges can also play a role in the deformation and failure process.

An analysis of the stability of these excavations depends primarily upon a correct interpretation of the structural geological conditions in the rock mass followed by a study



of the blocks and wedges which can be released by the creation of the excavation. Identification and visualisation of these blocks and wedges is by far the most important part of this analysis. Analysis of the stability of the blocks and wedges, and of the reinforcing forces required to stabilize them, is a relatively simple process once this identification has been carried out.

The Río Grande Pumped Storage Project is located in the Province of Córdoba in the Republic of Argentina. Four reversible pump-turbines operating at an average head of 170 m give the project a total installed capacity of 750 MW. These turbines are installed in a 25 m span, 50 m high, 105 m long cavern at an average depth of 160 m .

The rock in which the underground excavations are situated is a massive tonalitic gneiss of excellent quality (Amos et al, 1981). The gneiss has an average uniaxial compressive strength of 140 MPa. The maximum principal stress, determined by overcoring tests, is 9.4 MPa and is almost horizontal and oriented approximately normal to the cavern axis. In massive rocks, this 15:1 ratio of uniaxial strength to maximum principal stress is unlikely to result in any significant failure in the rock and this was confirmed by numerical stress analyses (Moretto, 1982). The principal type of instability which had to be dealt with in the underground excavations was that of potentially unstable blocks and wedges defined by intersecting structural features (Hammett and Hoek, 1981). In one section of the cavern, the axis of which is oriented in the direction 158-338, four joint sets were mapped and were found to have the following dip/dip direction values:

Table 5. Dip and dip direction values for joints in one location in the Río Grande cavern

N.	Dip	Dip dir.	Comments
1	50	131	infrequently occurring joints
2	85	264	shear joint set
3	70	226	shear joint set
4	50	345	tension joint set

Figure 4 is a perspective view of the Río Grande power cavern showing typical wedges which can be formed in the roof, sidewalls, bench and floor by joint sets 2, 3 and 4. These figures represent the maximum possible sizes of wedges which can be formed and, during construction, the sizes of the wedges were scaled down in accordance with average joint trace lengths measured in the excavation faces. In Figure 4 it is evident that the roof and the two sidewall wedges were potentially unstable and that they needed to be stabilised. This stabilisation was achieved by the placement of tensioned and grouted rockbolts which were installed at each stage of the cavern excavation. Decisions on the number, length and capacity of the rockbolts were made by on-site geotechnical staff using limit equilibrium calculations based upon the volume of the wedges defined by the measured trace lengths. For those wedges which involved sliding on one plane or along the line of intersection of two planes, rockbolts were installed across these planes to bring the sliding factor of safety of the wedge up to 1.5. For wedges which were free to fall from the roof, a factor of safety of 2 was used. This factor was calculated as the ratio

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of the total capacity of the bolts to the weight of the wedge and was intended to account for uncertainties associated with the bolt installation.

The floor wedge was of no significance while the wedges in the bench at the base of the upstream wall were stabilised by dowels placed in grout-filled vertical holes before excavation of the lower benches.

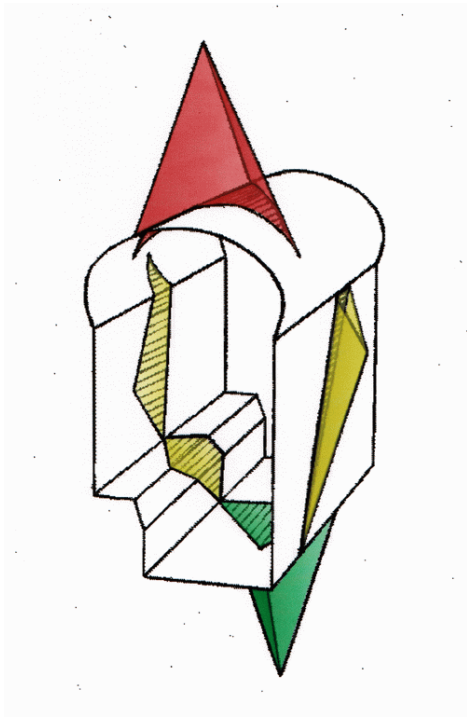


Figure 4: Perspective view of Río Grande power cavern showing potentially unstable wedges in the roof, sidewalls, bench and floor.

Early recognition of the potential instability problems, identification and visualization of the wedges which could be released and the installation of support at each stage of excavation, before the wedge bases were fully exposed, resulted in a very effective stabilisation program. Apart from a minimal amount of mesh and shotcrete applied to areas of intense jointing, no other support was used in the power cavern which has operated without any signs of instability since its completion in 1982.

### **Excavations in weak rock**

In contrast to the structurally controlled failures in strong rock discussed in the previous section, there are many cases where tunnels and caverns are excavated in rock masses which are weak as a result of intense jointing or because the rock material itself has a low strength. Rocks such as shales, mudstones, siltstones, phyllites and tuffs are typical weak rocks in which even moderate in situ stresses are likely to induce failure in the rock surrounding underground excavations.

Progressive failure of this type, which can occur in the rock surrounding an underground excavation in a weak rock mass, is a difficult analytical problem and there are no simple numerical models nor factor of safety calculations which can be used to define acceptable limits to this failure process. Judgement on the adequacy of a support design has to be based upon an evaluation of a number of factors such as the magnitude and distribution of deformations in the rock and the stresses induced in support elements such as grouted cables, steel sets or concrete linings. This design process is illustrated by means of an example.

The Mingtan pumped storage project is located in the central region of the island of Taiwan and utilizes the 400 m head difference between the Sun Moon Lake and the Shuili River to generate up to 1600 MW at times of peak demand. The power cavern is 22 m wide, 46 m high and 158 m long and a parallel transformer hall is 13 m wide, 20 m high and 17 m long. The caverns are 45 m apart and are located at a depth of 30 m below surface in the steep left bank of the Shuili river (Liu, Cheng and Chang, 1988).

The rock mass consists of weathered, interbedded sandstones, siltstones and shales dipping at about 35° to the horizontal. The Rock Mass Ratings (RMR) (Bieniawski, 1974) and Tunnelling Quality Index Q (Barton, Lien and Lunde, 1974) and approximate shear strength values for the various components of the rock mass are given in Table 6 below.

Table 6. Rock mass classifications and approximate friction angles  $\phi$  and cohesive strengths  $c$  for the rock mass in which the Mingtan power cavern is excavated

Rock type	RMR	Q	$\phi$ degrees	$c'$ MPa
Jointed sandstone	63-75	12-39	50	1.0
Bedded sandstone	56-60	7-31	45	0.8
Faults or shears	10-33	0.1-1.1	30-40	0.15-0.3

Weak beds of siltstone, up to 2 m thick, appear to have caused a concentration of shear movements during tectonic activity so that fault zones have developed parallel to the bedding. The common feature observed for all these faults is the presence of continuous clay filling with a thickness varying from a few mm to 200 mm. The cavern axis is intentionally oriented at right angles to the strike of these faults.

The measured in situ stresses in the rock mass surrounding the cavern are approximately

$$\begin{aligned}\text{Maximum principal stress (horizontal)} & \quad \sigma_{\max} = 10.9 \text{ MPa} \\ \text{Minimum principal stress (vertical)} & \quad \sigma_{\min} = 7.5 \text{ MPa}\end{aligned}$$

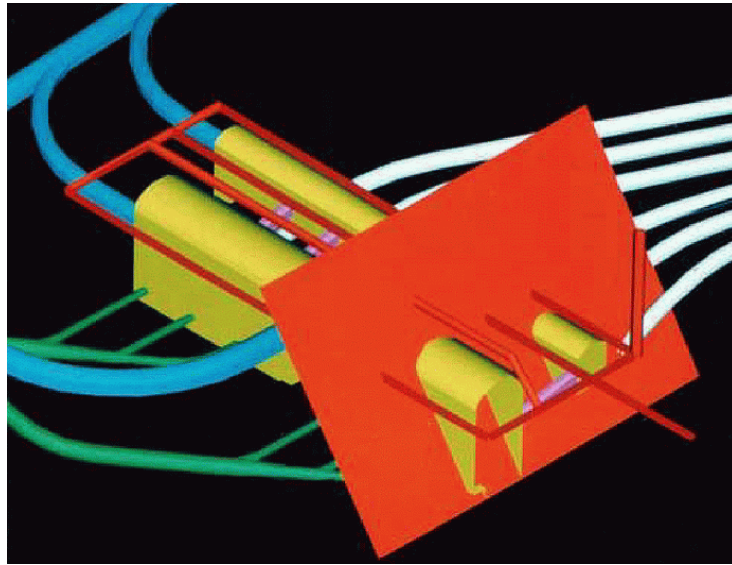


Figure 5: Orientation of the underground excavations in relation to the faults in the bedded sandstone surrounding the power cavern and transformer hall of the Mingtan Project. The red plane indicates the dip and strike of the faults.

Bedding faults of significant thickness which were intersected in the roof of the cavern were treated by using high pressure water jets to remove the clay and then filling the cavities with non shrink cementitious mortar (Cheng, 1987, Moy and Hoek, 1989). This was followed by the installation of 50 tonne capacity untensioned grouted cables from a drainage gallery 10 m above the cavern roof in order to create a pre-reinforced rock mass above the cavern. All of this work was carried out from construction adits before the main contract for the cavern excavation commenced.

The initial design of the reinforcing cables was based upon experience and precedent practice. Figures 6 and 7 give the lengths of rockbolts and cables in the roof and sidewalls of some typical large powerhouse caverns in weak rock masses. Plotted on the same graphs are empirical relationships suggested by Barton (1989) for bolt and cable lengths for underground powerhouses.

During benching down in the cavern, 112 tonne capacity tensioned and grouted cables were installed on a 3 m x 3 m grid in the sidewalls. The final layout of the cables in the rock surrounding the power cavern and the transformer hall is illustrated in Figure 8. Five metre long grouted rockbolts were installed as required at the centre of the squares formed by the cable face plates. A 50 mm layer of steel fibre reinforced microsilica shotcrete was applied within 5 to 10 m of the face. This shotcrete was later built up to a thickness of 150 mm on the roof and upper sidewalls and 50 mm on the lower sidewalls where it would eventually be incorporated into the concrete foundations.



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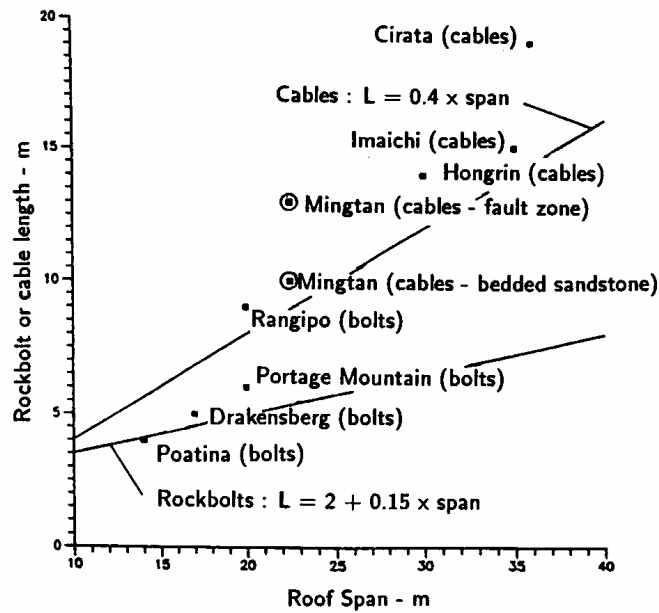


Figure 6: Lengths of rockbolts and cables used for roof support in some large caverns in weak rock. Equations defining trend lines were suggested by Barton (1989).

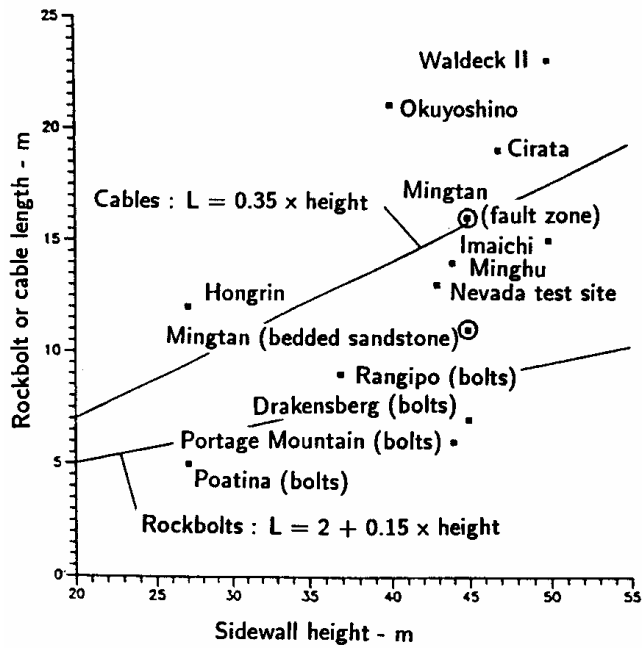


Figure 7: Lengths of rockbolts and cables used for sidewall support in some large caverns in weak rock. Equations defining trend lines were suggested by Barton (1989).

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A key element in the decision making process on the adequacy of the support system was a monitoring and analysis process which involved the following steps :

1. Displacements in the rock surrounding the excavations monitored by means of convergence arrays and extensometers, some of which had been installed from construction galleries before excavation of the caverns commenced.
2. Numerical modelling of each excavation stage using non-linear multiple-material models. The material properties used in the models of the early excavation stages were adjusted to obtain the best match between predicted and measured displacements.
3. Prediction of displacements and support loads during future excavation stages and adjustment of support capacity, installation and pre-tensioning to control displacements and cable loads.
4. Measurement of displacements and cable loads (using load cells on selected cables which had been de-bonded) and comparison between measured and predicted displacements and cable loads.
5. Installation of additional cables or adjustment of cable loads to control unusual displacements or support loads.

The aim of this program was to maintain as uniform a displacement pattern around the excavations as possible and to keep the loads on the cables at less than 45% of their yield load. The intermediate rockbolts and the shotcrete were not accounted for in the numerical modelling since it was assumed that their role was confined to supporting the rock immediately adjacent to the excavations and that the overall stability was controlled by the 10 to 15 m long grouted cables.

Figure 8 shows the combination of materials used in analysing one section of the cavern, assuming that the bedding faults could be represented by horizontal layers in the two-dimensional model. In order to match the measured and predicted displacements in the rock mass, it was found that a 2.5 m thick zone of softened and weakened material had to be wrapped around the excavations to account for blast damaged material (achieving good blasting results was difficult in this interbedded rock).

In Figure 9, the predicted and measured displacements along six extensometers installed in the power cavern sidewalls are compared. The overall agreement is considered to be acceptable. Maximum sidewall displacements were of the order of 100 mm at the mid-height of the upstream wall, adjacent to one of the major faults. Elsewhere, displacements were of the order to 25 to 46 mm.

Figure 10 shows the results of monitoring at seven stations along the axis of the power cavern. Before excavation of the cavern commenced, extensometers were installed at

each of these stations from a drainage gallery above the roof arch and from construction galleries as shown in the upper part of Figure 10. In addition, load cells were installed on cables adjacent to some of the extensometers.

Rapid responses were recorded in all extensometers and load cells as the top heading passed underneath them. Further responses occurred as the haunches of the cavern arch were excavated and as the first bench was removed. As can be seen from the plots, after this rapid response to the initial excavation stages, the displacements and cable loads became stable and showed very little tendency to increase with time. The difference in the magnitudes of the displacements and cable loads at different stations can be related to the proximity of the monitoring instruments to faults in the rock above the cavern arch.

The rapid load acceptance and the modest loading of the cables together with the control of the displacements in the rock mass were the goals of the support design. Measurements obtained from the extensometers and cable load cells indicate that these goals have been met.

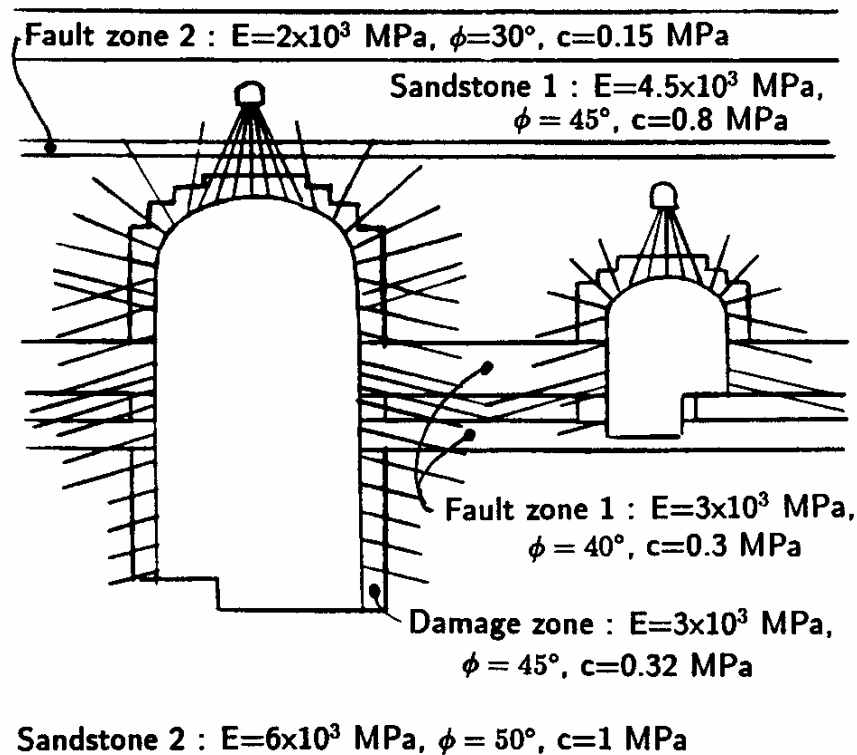


Figure 8: Layout of cables used to support the rock surrounding the power cavern and the transformer hall in the Mingtan pumped storage project. The location and properties of the rock units represent those used in the numerical analysis of failure, deformation and cable loading in a typical vertical section.

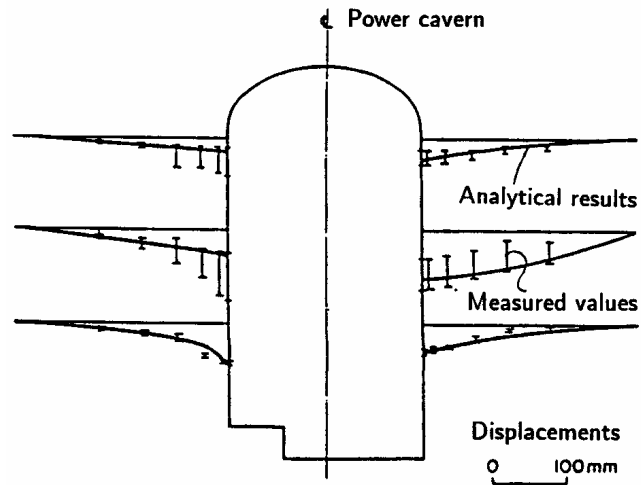


Figure 9: Comparison between calculated and measured displacements along six extensometers installed in the sidewalls of the Mingtan power cavern.

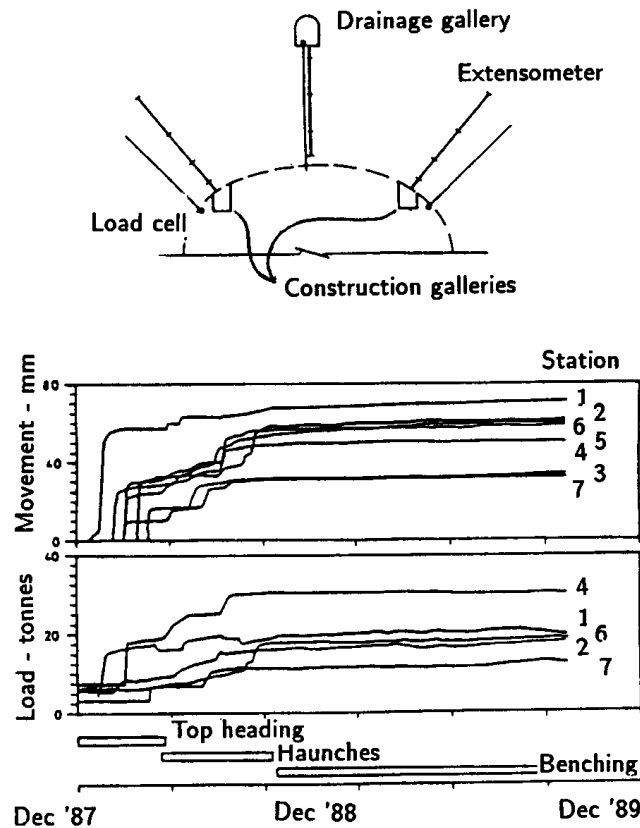


Figure 10: Surface displacements and cable loads measured at seven stations along the power cavern axis.



## **Factor of safety**

The four case histories, discussed in previous sections, have been presented to demonstrate that a variety of criteria have to be considered in deciding upon the adequacy of a rock structure to perform its design objectives. This is true for any design in rock since the performance of each structure will be uniquely dependent upon the particular set of rock conditions, design loads and intended end use.

In one group of structures, traditional designs have been based upon a 'factor of safety' against sliding. These structures, which include gravity and fill dams as well as rock and soil slopes, all involve the potential for sliding along well defined failure surfaces. The factor of safety is defined as the factor by which the shear strength parameters may be reduced in order to bring the slope (or dam foundation) into a state of limiting equilibrium (Morgenstern, 1991). The numerical value of the factor of safety chosen for a particular design depends upon the level of confidence which the designer has in the shear strength parameters, the groundwater pressures, the location of the critical failure surface and the magnitude of the external driving forces acting upon the structure.

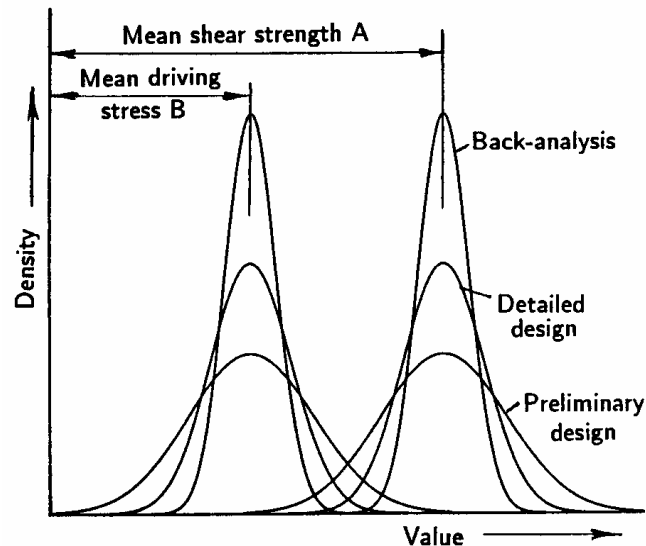


Figure 11: Hypothetical distribution curves representing the degree of uncertainty associated with information on driving stresses and shear strengths at different stages in the design of a structure such as a dam foundation.

Figure 11 illustrates a set of hypothetical distribution curves representing the degree of uncertainty associated with available information on shear strength parameters and disturbing stresses for different stages in the design of a rock or soil structure. The factor of safety is defined as  $A/B$  where  $A$  is the mean of the distribution of shear strength

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values and  $B$  is the mean of the distribution of driving stresses. For the purpose of this discussion, the same factor of safety has been assumed for all three cases illustrated.

During preliminary design studies, the amount of information available is usually very limited. Estimates of the shear strength of the rock or soil are generally based upon the judgement of an experienced engineer or geologist which may be supplemented, in some cases, by estimates based upon rock mass classifications or simple index tests. Similarly, the disturbing forces are not known with very much certainty since the location of the critical failure surface will not have been well defined and the magnitude of externally applied loads may not have been established. In the case of dam design, the magnitude of the probable maximum flood, which is usually based upon probabilistic analysis, frequently remains ill defined until very late in the design process.

For this case, the range of both available shear strength and disturbing stresses, which have to be considered, is large. If too low a factor of safety is used, there may be a significant probability of failure, represented by the section where the distribution curves overlap in Figure 11. In order to minimise this failure probability, a high value for the factor of safety is sometimes used. For example, in the 1977 edition of the US Bureau of Reclamation Engineering Monograph on Design Criteria for Concrete Arch and Gravity Dams, a factor of safety of 3.0 is recommended for normal loading conditions when 'only limited information is available on the strength parameters'. This value can be reduced to 2.0 when the strength parameters are 'determined by testing of core samples from a field investigation program or by past experience'.

During detailed design studies, the amount of information available is usually significantly greater than in the preliminary design stage discussed above. A comprehensive program of site investigations and laboratory or in situ shear strength tests will normally have been carried out and the external loads acting on the structure will have been better defined. In addition, studies of the groundwater flow and pressure distributions in the rock mass, together with modifications of these distributions by grouting and drainage, will usually have been carried out. Consequently, the ranges of shear strength and driving stress values, which have to be considered in the design, are smaller and the distribution curves are more tightly constrained.

The case histories of the Downie Slide and Dutchman's Ridge, discussed earlier, are good examples of designs based upon back-analyses. In both of these cases, very extensive site investigations and displacement monitoring had established the location of the critical failure surfaces with a high degree of certainty. Careful monitoring of the groundwater in the slopes (256 piezometer measuring points were installed in Dutchman's Ridge) had defined the water pressures in the slopes and their fluctuations over several years. Some shear testing on fault material recovered from cores was carried out but, more importantly, the mobilized shear strength along the potential failure surfaces was calculated by back-analysis, assuming a factor of safety of 1.00 for existing conditions.

Figure 11 illustrates the hypothetical distribution curves for the range of values for shear strength and driving stresses for the case of a structure in which an existing failure has been carefully back-analyzed. Depending upon the degree of care which has been taken with this back-analysis, these curves will be very tightly constrained and a low factor of safety can be used for the design of the remedial works.

This discussion illustrates the point that different factors of safety may be appropriate for different stages in the design of a rock structure. This difference is primarily dependent upon the level of confidence which the designer has in the values of shear strength to be included in the analysis. Hence, a critical question which arises in all of these cases is the determination or estimation of the shear strength along the potential sliding surface. In a paper on the strength of rockfill materials, Marachi, Chan and Seed (1972) summarize this problem as follows: 'No stability analysis, regardless of how intricate and theoretically exact it may be, can be useful for design if an incorrect estimation of the shearing strength of the construction material has been made'.

Except in simple cases involving homogeneous soils or planar continuous weak seams, determination of the shear strength along potential sliding surfaces is a notoriously difficult problem. This is particularly true of the determination of the cohesive component,  $c'$ , of the commonly used Mohr-Coulomb failure criterion. Laboratory test specimens tend to be too small to give representative results while in situ tests are difficult and expensive and, unless carried out with very great care, are liable to give unreliable results.

Table 7: Factors of safety for different loading in the design of earth and rockfill dams.

Loading condition	S.F.	Remarks
End of construction porewater pressures in the dam and undissipated porewater pressures in the foundation. No reservoir loading.	1.3	
Reservoir at full supply level with steady state seepage in the dam and undissipated end-of-construction porewater pressures in the foundation.	1.3	Possibly the most critical (even if rare) condition.
Reservoir at full supply level with steady state seepage.	1.5	Critical to design.
Reservoir at probable maximum flood level with steady state seepage conditions.	1.2	
Rapid reservoir drawdown from full supply level to minimum supply level	1.3	Not significant in design. Failures very rare and, if they occur, usually shallow.

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For failure surfaces which involve sliding on rough or undulating rock surfaces such as joints or bedding planes, the methodology proposed by Barton (1976) is appropriate for estimating the overall shear strength of the potential sliding surface. This involves adding a measured or estimated roughness component to the basic frictional strength which can be determined on sawn and polished laboratory shear test specimens.

For heavily jointed rock masses in which there are no dominant weakness zones such as faults or shear zones, a crude estimate of the shear strength of the rock mass can be obtained by means of the use of rock mass classification systems as proposed by Hoek and Brown (1988).

In all cases, a greater reliance can be placed upon the frictional component,  $\phi$ , of the Mohr-Coulomb shear strength equation and extreme care has to be taken in the estimation of the cohesive strength,  $c'$ . Where no reliable estimates of this value are available from carefully conducted shear tests or from back-analysis of existing failures, it is prudent to assume a cohesive strength of zero for any stability analysis involving structures such as dam foundations.

In the design of fill and gravity dams there is a tendency to move away from the high factors of safety of 2 or 3 which have been used in the past, provided that care is taken in choosing sensible conservative shear strength parameters, particularly for continuous weak seams in the foundations. An example of the range of factors of safety which can be used in the design of earth or rockfill dams is given in Table 7.

### **Probabilistic analyses**

The uncertainty associated with the properties of geotechnical materials and the great care which has to be taken in selecting appropriate values for analyses has prompted several authors to suggest that the traditional deterministic methods of slope stability analyses should be replaced by probabilistic methods (Priest and Brown, 1983, McMahon, 1975, Vanmarcke, 1980, Morriss and Stoter, 1983, Read and Lye, 1983).

One branch of rock mechanics in which probabilistic analyses have been accepted for many years is that of the design of open pit mine slopes. This is because open pit planners are familiar with the concepts of risk analysis applied to ore grade and metal price fluctuations. Probabilistic methods are used in estimating the economic viability of various options in developing an open pit mine and so it is a small step to incorporate the probability of a geotechnical failure into the overall risk assessment of the mine. The mine planner has the choice of reducing the probability of failure by the installation of reinforcement, reducing the angle of the slope or accepting that failure will occur and providing for extra equipment which may be needed to clean up the failure. Since the mine is usually owned and operated by a single company and access to the mine benches is restricted to trained personnel, accepting a risk of failure and dealing with the consequences on a routine basis is a viable option.



On the other hand, the emotional impact of suggesting to the public that there is a finite risk of failure attached to a dam design is such that it is difficult to suggest the replacement of the standard factor of safety design approach with one which explicitly states a probability of failure or a coefficient of reliability. The current perception is that the factor of safety is more meaningful than the probability of failure. Even if this were not so, there is still the problem of deciding what probability of failure is acceptable for a rock structure to which the general public has access.

In spite of these difficulties, there does appear to be a slow but steady trend in society to accept the concepts of risk analysis more readily than has been the case in the past. The geotechnical community has an obligation to take note of these developments and to encourage the teaching and practical use of probabilistic as well as deterministic techniques with the aim of removing the cloak of mystery which surrounds the use of these methods.

Fortunately, there is a compromise solution which is a form of risk analysis used intuitively by most experienced engineers. This is a parametric analysis in which a wide range of possibilities are considered in a conventional deterministic analysis in order to gain a 'feel' for the sensitivity of the design. Hence, the factor of safety for a slope would be calculated for both fully drained and fully saturated groundwater conditions, for a range of friction angles and cohesive strengths covering the full spectrum which could be anticipated for the geological conditions existing on the site, for external forces ranging from zero to the maximum possible for that slope. The availability of user-friendly microcomputer software for most forms of limit equilibrium analysis means that these parametric studies can be carried out quickly and easily for most designs.

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# Factor of safety and probability of failure

## Introduction

How does one assess the acceptability of an engineering design? Relying on judgement alone can lead to one of the two extremes illustrated in Figure 1. The first case is economically unacceptable while the example illustrated in the drawing on the right violates all normal safety standards.

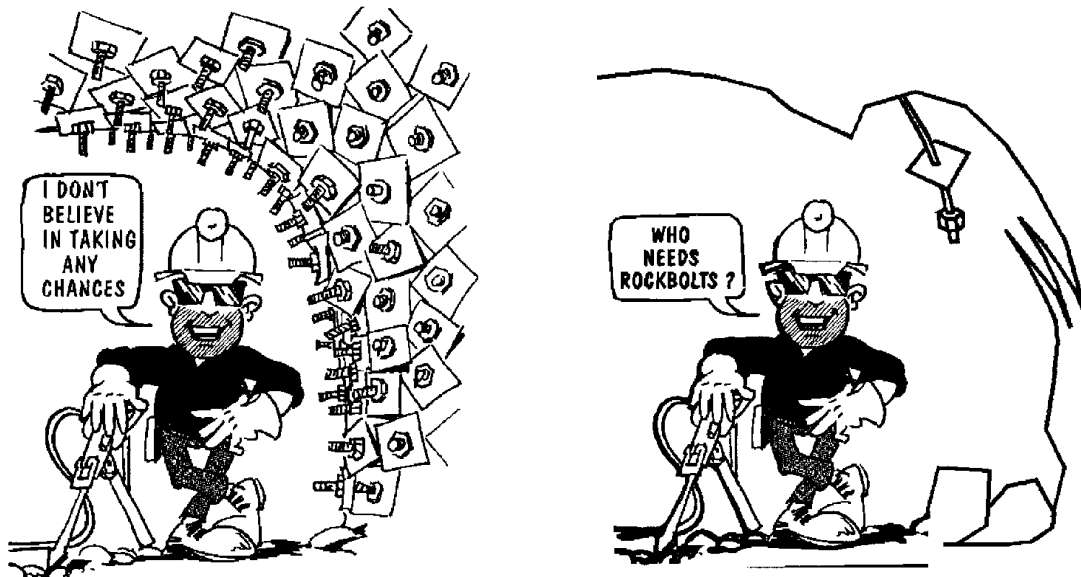


Figure 1: Rockbolting alternatives involving individual judgement. (Drawings based on a cartoon in a brochure on rockfalls published by the Department of Mines of Western Australia.)

## Sensitivity studies

The classical approach used in designing engineering structures is to consider the relationship between the capacity  $C$  (strength or resisting force) of the element and the demand  $D$  (stress or disturbing force). The Factor of Safety of the structure is defined as  $F = C/D$  and failure is assumed to occur when  $F$  is less than unity.

## *Factor of safety and probability of failure*

Rather than base an engineering design decision on a single calculated factor of safety, an approach which is frequently used to give a more rational assessment of the risks associated with a particular design is to carry out a sensitivity study. This involves a series of calculations in which each significant parameter is varied systematically over its maximum credible range in order to determine its influence upon the factor of safety.

This approach was used in the analysis of the Sau Mau Ping slope in Hong Kong, described in detail in another chapter of these notes. It provided a useful means of exploring a range of possibilities and reaching practical decisions on some difficult problems. On the following pages this idea of sensitivity studies will be extended to the use of probability theory and it will be shown that, even with very limited field data, practical, useful information can be obtained from an analysis of probability of failure.

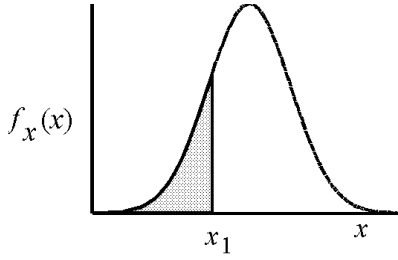
### **An introduction to probability theory**

A complete discussion on probability theory exceeds the scope of these notes and the techniques discussed on the following pages are intended to introduce the reader to the subject and to give an indication of the power of these techniques in engineering decision making. A more detailed treatment of this subject will be found in a book by Harr (1987) entitled 'Reliability-based design in civil engineering'. A paper on geotechnical applications of probability theory entitled 'Evaluating calculated risk in geotechnical engineering' was published by Whitman (1984) and is recommended reading for anyone with a serious interest in this subject. Pine (1992), Tyler et al (1991), Hatzor and Goodman (1993) and Carter (1992) have published papers on the application of probability theory to the analysis of problems encountered in underground mining and civil engineering.

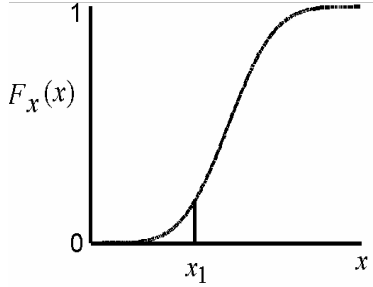
Most geotechnical engineers regard the subject of probability theory with doubt and suspicion. At least part of the reason for this mistrust is associated with the language which has been adopted by those who specialise in the field of probability theory and risk assessment. The following definitions are given in an attempt to dispel some of the mystery which tends to surround this subject.

*Random variables:* Parameters such as the angle of friction of rock joints, the uniaxial compressive strength of rock specimens, the inclination and orientation of discontinuities in a rock mass and the measured in situ stresses in the rock surrounding an opening do not have a single fixed value but may assume any number of values. There is no way of predicting exactly what the value of one of these parameters will be at any given location. Hence, these parameters are described as random variables.





Probability density function (PDF)



Cumulative distribution function (CDF)

*Probability distribution:* A probability density function (PDF) describes the relative likelihood that a random variable will assume a particular value. A typical probability density function is illustrated opposite. In this case the random variable is continuously distributed (i.e., it can take on all possible values). The area under the PDF is always unity.

An alternative way of presenting the same information is in the form of a cumulative distribution function (CDF), which gives the probability that the variable will have a value less than or equal to the selected value. The CDF is the integral of the corresponding probability density function, i.e., the ordinate at  $x_1$  on the cumulative distribution is the area under the probability density function to the left of  $x_1$ . Note the  $f_x(x)$  is used for the ordinate of a PDF while  $F_x(x)$  is used for a CDF.

One of the most common graphical representations of a probability distribution is a histogram in which the fraction of all observations falling within a specified interval is plotted as a bar above that interval.

*Data analysis:* For many applications it is not necessary to use all of the information contained in a distribution function and quantities summarised only by the dominant features of the distribution may be adequate.

*The sample mean or expected value* or first moment indicates the centre of gravity of a probability distribution. A typical application would be the analysis of a set of results  $x_1, x_2, \dots, x_n$  from uniaxial strength tests carried out in the laboratory. Assuming that there are  $n$  individual test values  $x_i$ , the mean  $\bar{x}$  is given by:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (1)$$

*The sample variance  $s^2$  or the second moment* about the mean of a distribution is defined as the mean of the square of the difference between the value of  $x_i$  and the mean value  $\bar{x}$ .

### *Factor of safety and probability of failure*

Hence:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (2)$$

Note that, theoretically, the denominator for calculation of variance of samples should be  $n$ , not  $(n - 1)$ . However, for a finite number of samples, it can be shown that the correction factor  $n/(n-1)$ , known as Bessel's correction, gives a better estimate. For practical purposes the correction is only necessary when the sample size is less than 30.

The *standard deviation*  $s$  is given by the positive square root of the variance  $s^2$ . In the case of the commonly used normal distribution, about 68% of the test values will fall within an interval defined by the *mean  $\pm$  one standard deviation* while approximately 95% of all the test results will fall within the range defined by the *mean  $\pm$  two standard deviations*. A small standard deviation will indicate a tightly clustered data set while a large standard deviation will be found for a data set in which there is a large scatter about the mean.

The *coefficient of variation* (COV) is the ratio of the standard deviation to the mean, i.e.  $COV = s/\bar{x}$ . COV is dimensionless and it is a particularly useful measure of uncertainty. A small uncertainty would typically be represented by a  $COV = 0.05$  while considerable uncertainty would be indicated by a  $COV = 0.25$ .

*Normal distribution:* The *normal* or *Gaussian* distribution is the most common type of probability distribution function and the distributions of many random variables conform to this distribution. It is generally used for probabilistic studies in geotechnical engineering unless there are good reasons for selecting a different distribution. Typically, variables which arise as a sum of a number of random effects, none of which dominate the total, are normally distributed.

The problem of defining a normal distribution is to estimate the values of the governing parameters which are the true mean ( $\mu$ ) and true standard deviation ( $\sigma$ ). Generally, the best estimates for these values are given by the sample mean and standard deviation, determined from a number of tests or observations. Hence, from equations 1 and 2:

$$\mu = \bar{x} \quad (3)$$

$$\sigma = s \quad (4)$$

It is important to recognise that equations 3 and 4 give the most probable values of  $\mu$  and  $\sigma$  and not necessarily the true values.

### *Factor of safety and probability of failure*

Obviously, it is desirable to include as many samples as possible in any set of observations but, in geotechnical engineering, there are serious practical and financial limitations to the amount of data which can be collected. Consequently, it is often necessary to make estimates on the basis of judgement, experience or from comparisons with results published by others. These difficulties are often used as an excuse for not using probabilistic tools in geotechnical engineering but, as will be shown later in this chapter, useful results can still be obtained from very limited data.

Having estimated the mean  $\mu$  and standard deviation  $\sigma$ , the probability density function for a normal distribution is defined by:

$$f_x(x) = \frac{\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]}{\sigma\sqrt{2\pi}} \quad (5)$$

for  $-\infty \leq x \leq \infty$ .

As will be seen later, this range of  $-\infty \leq x \leq \infty$  can cause problems when a normal distribution is used as a basis for a Monte Carlo analysis in which the entire range of values is randomly sampled. This can give rise to a few very small numbers (sometimes negative) and very large numbers which, in certain analyses, can cause numerical instability. In order to overcome this problem the normal distribution is sometimes truncated so that only values falling within a specified range are considered valid.

There is no closed form solution for the cumulative distribution function (CDF) which must be found by numerical integration.

*Other distributions:* In addition to the commonly used normal distribution there are a number of alternative distributions which are used in probability analyses. Some of the most useful are:

*Beta distributions* (Harr, 1987) are very versatile distributions which can be used to replace almost any of the common distributions and which do not suffer from the extreme value problems discussed above because the domain (range) is bounded by specified values.

*Exponential distributions* are sometimes used to define events such as the occurrence of earthquakes or rockbursts or quantities such as the length of joints in a rock mass.

*Lognormal distributions* are useful when considering processes such as the crushing of aggregates in which the final particle size results from a number of collisions of particles of many sizes moving in different directions with different velocities. Such

### *Factor of safety and probability of failure*

multiplicative mechanisms tend to result in variables which are lognormally distributed as opposed to the normally distributed variables resulting from additive mechanisms.

*Weibul distributions* are used to represent the lifetime of devices in reliability studies or the outcome of tests such as point load tests on rock core in which a few very high values may occur.

It is no longer necessary for the person starting out in the field of probability theory to know and understand the mathematics involved in all of these probability distributions since commercially available software programs can be used to carry out many of the computations automatically. Note that the author is not advocating the blind use of 'black-box' software and the reader should exercise extreme caution is using such software without trying to understand exactly what the software is doing. However there is no point in writing reports by hand if one is prepared to spend the time learning how to use a good word-processor correctly and the same applies to mathematical software.

One of the most useful software packages for probability analysis is a Microsoft Excel add-in program called @RISK<sup>1</sup> which can be used for risk evaluations using the techniques described below.

*Sampling techniques:* Consider a problem in which the factor of safety depends upon a number of random variables such as the cohesive strength  $c$ , the angle of friction  $\phi$  and the acceleration  $\alpha$  due to earthquakes or large blasts. Assuming that the values of these variables are distributed about their means in a manner which can be described by one of the continuous distribution functions such as the normal distribution described earlier, the problem is how to use this information to determine the distribution of factor of safety values and the probability of failure.

The Monte Carlo method uses random or pseudo-random numbers to sample from probability distributions and, if sufficiently large numbers of samples are generated and used in a calculation such as that for a factor of safety, a distribution of values for the end product will be generated. The term 'Monte Carlo' is believed to have been introduced as a code word to describe this hit-and-miss technique used during secret work on the development of the atomic bomb during World War II (Harr 1987). Today, Monte Carlo techniques can be applied to a wide variety of problems involving random behaviour and a number of algorithms are available for generating random Monte Carlo samples from different types of input probability distributions. With highly optimised software programs such as @RISK, problems involving relatively large samples can be run efficiently on most desktop or portable computers.

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<sup>1</sup> @RISK is available from [www.palisade.com](http://www.palisade.com).

### *Factor of safety and probability of failure*

The *Latin Hypercube* sampling technique (Imam et al, 1980, Startzman and Watterbarger, 1985) is a relatively recent development which gives comparable results to the Monte Carlo technique but with fewer samples. The method is based upon stratified sampling with random selection within each stratum. Typically an analysis using 1000 samples obtained by the Latin Hypercube technique will produce comparable results to an analysis using 5000 samples obtained using the Monte Carlo method. Both techniques are incorporated in the program @RISK.

Note that both the Monte Carlo and the Latin Hypercube techniques require that the distribution of all the input variables should either be known or that they be assumed. When no information on the distribution is available it is usual to assume a normal or a truncated normal distribution.

The *Generalised Point Estimate Method*, developed by Rosenbleuth (1981) and discussed in detail by Harr (1987), can be used for rapid calculation of the mean and standard deviation of a quantity such as a factor of safety which depends upon random behaviour of input variables. Hoek (1989) discussed the application of this technique to the analysis of surface crown pillar stability while Pine (1992) has applied this technique to the analysis of slope stability and other mining problems.

To calculate a quantity such as a factor of safety, two point estimates are made at one standard deviation on either side of the mean ( $\mu \pm \sigma$ ) from each distribution representing a random variable. The factor of safety is calculated for every possible combination of point estimates, producing  $2^n$  solutions where  $n$  is the number of random variables involved. The mean and the standard deviation of the factor of safety are then calculated from these  $2^n$  solutions.

While this technique does not provide a full distribution of the output variable, as do the Monte Carlo and Latin Hypercube methods, it is very simple to use for problems with relatively few random variables and is useful when general trends are being investigated. When the probability distribution function for the output variable is known, for example, from previous Monte Carlo analyses, the mean and standard deviation values can be used to calculate the complete output distribution.

Some of the techniques described above have been incorporated into specialized commercial software packages and one of these called RocPlane<sup>2</sup> will be used to analyse the Sau Mau Ping slope.

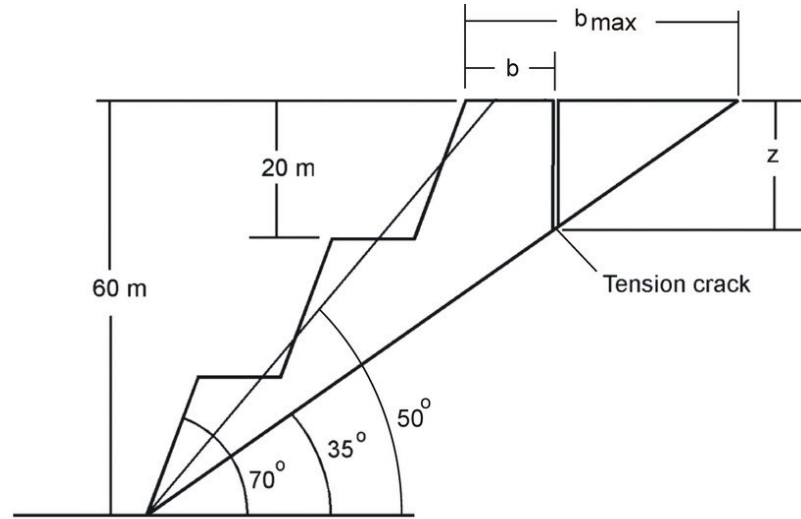
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<sup>2</sup> Available from [www.rocscience.com](http://www.rocscience.com)



### Probability of failure

In the case of the Sau Mau Ping slope problem the input parameters and assumed distributions for the calculation of the factor of safety of the overall slope with a tension crack are as follows:



#### 1. Fixed dimensions:

Overall slope height	$H = 60\text{ m}$
Overall slope angle	$\psi_f = 50^\circ$
Failure plane angle	$\psi_p = 35^\circ$
Upper slope inclination	horizontal
Bench width $b_{\text{max}} = H(\cot \psi_p - \cot \psi_f)$	$b_{\text{max}} = 35.34\text{ m}$
Unit weight of rock	$\gamma_r = 2.6\text{ tonnes/m}^3$
Unit weight of water	$\gamma_w = 1.0\text{ tonnes/m}^3$

#### 2. Random variables

	Mean values	Standard deviation	Distribution
Friction angle on joint surface	$\phi = 35^\circ$	$\pm 5$	Normal
Cohesive strength of joint surface	$c = 10\text{ tonnes/m}^2$	$\pm 2$	Normal
Depth of tension crack	$z = 14\text{ m}$	$\pm 3$	Normal
Distance from crest to tension crack	$b = 15.3\text{ m}$	$\pm 4$	Normal
Depth of water in tension crack	$z_w = z/2$ min = 0, max = z		Exponential
Ratio of horizontal earthquake to gravitational acceleration	$\alpha = 0.08$ min = 0, max = $2\alpha$		Exponential

### *Factor of safety and probability of failure*

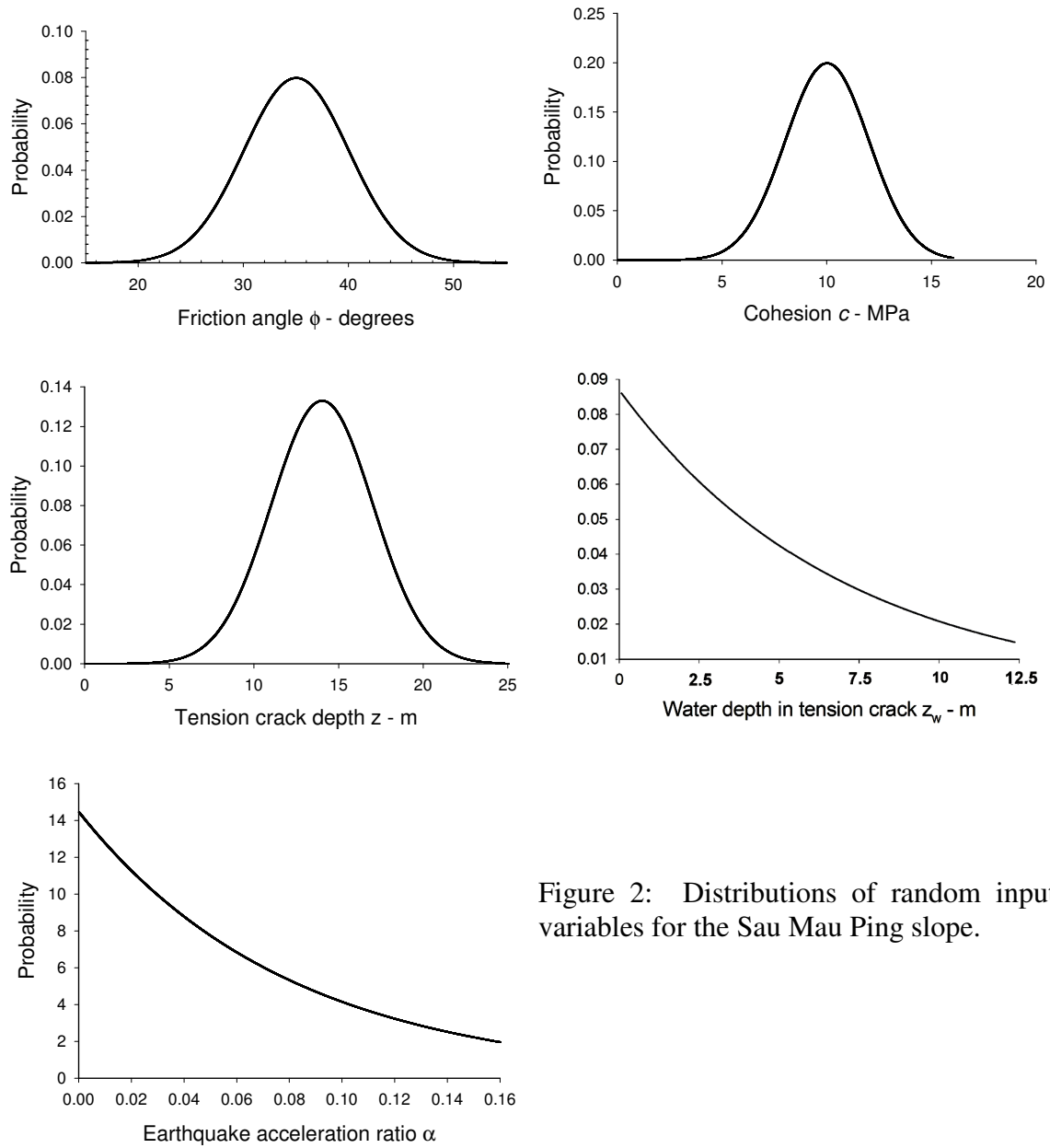


Figure 2: Distributions of random input variables for the Sau Mau Ping slope.

Figure 2 illustrates the plots of the probability distribution functions of the random input variables. It is worth discussing each of the plots in detail to demonstrate the reasoning behind the choice of the probability distribution functions.

### *Factor of safety and probability of failure*

*Friction angle  $\phi$*  - A truncated normal distribution has been assumed for this variable. The mean is assumed to be  $35^\circ$  which is the approximate centre of the assumed shear strength range illustrated in Figure 8 of “A slope stability problem in Hong Kong”. The standard deviation of  $5^\circ$  implies that about 68% of the friction angle values defined by the distribution will lie between  $30^\circ$  and  $40^\circ$ . The normal distribution is truncated by a minimum value of  $15^\circ$  and a maximum value of  $70^\circ$  which have been arbitrarily chosen as the extreme values represented by a smooth slickensided surface and a fresh, rough tension fracture.

*Cohesive strength  $c$*  - Again using the assumed range of shear strength values illustrated in Figure 8 of “A slope stability problem in Hong Kong”, a value of 10 tonnes/m<sup>2</sup> has been chosen as the mean cohesive strength and the standard deviation has been set at 2 tonnes/m<sup>2</sup> on the basis of this diagram. In order to allow for the wide range of possible cohesive strengths the minimum and maximum values used to truncate the normal distribution are 0 and 25 tonnes/m<sup>2</sup> respectively. Those with experience in the interpretation of laboratory shear strength test results may argue that the friction angle  $\phi$  and the cohesive strength  $c$  are not independent variables as has been assumed in this analysis. This is because the cohesive strength generally drops as the friction angle rises and vice versa. The program @RISK allows the user to define variables as dependent but, for the sake of simplicity, the friction angle  $\phi$  and the cohesive strength  $c$  have been kept independent for this analysis.

*Distance of tension crack behind face  $b$*  - The program RocPlane uses the horizontal distance  $b$  of the tension crack behind the slope crest as input in place of the tension crack depth  $z$  because  $b$  can be measured in the field and also because it is not influenced by the inclination of the upper slope. Hoek and Bray (1974) give the value of  $b$  as  $b = H(\sqrt{\cot \psi_f \tan \psi_p} - \cot \psi_f)$  with the limits as  $0 < b < H(\cot \psi_p - \cot \psi_f)$ .

*Tension crack depth  $z$*  - Equation 6 in “A slope stability problem in Hong Kong”, defining the tension crack depth, has been derived by minimisation of equation 5 in that chapter. For the purposes of this analysis it has been assumed that this value of  $z$  (14 m for the assumed conditions) represents the mean tension crack depth. A truncated normal distribution is assumed to define the possible range of tension crack depths and the standard deviation has been arbitrarily chosen at 3 m. The minimum tension crack depth is zero but a value of 0.1 m has been chosen to avoid possible numerical problems. The maximum tension crack depth is given by  $z = H(1 - \tan \psi_p / \tan \psi_f) = 24.75$  m which occurs when the vertical tension crack is located at the crest of the slope.

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*Water depth  $z_w$  in tension crack* - The water which would fill the tension crack in this slope would come from direct surface run-off during heavy rains. In Hong Kong the heaviest rains occur during typhoons and it is likely that the tension crack would be completely filled during such events. The probability of occurrence of typhoons has been defined by a truncated exponential distribution where the mean water depth is assumed to be one half the tension crack depth. The maximum water depth cannot exceed the tension crack depth  $z$  and, as defined by the exponential distribution, this value would occur very rarely. The minimum water depth is zero during dry conditions and this is assumed to be a frequent occurrence.

*Ratio of horizontal earthquake acceleration to gravitational acceleration  $\alpha$*  - The frequent occurrence of earthquakes of different magnitudes can be estimated by means of an exponential distribution which suggests that large earthquakes are very rare while small ones are very common. In the case of Hong Kong local wisdom suggested a 'design' horizontal acceleration of 0.08g. In other words, this level of acceleration could be anticipated at least once during the operating life of a civil engineering structure. A rough rule of thumb suggests that the 'maximum credible' acceleration is approximately twice the 'design' value. Based upon these very crude guidelines, the distribution of values of  $\alpha$  used in these calculations was defined by a truncated exponential distribution with a mean value of  $\alpha = 0.08$ , a maximum of 0.16 and a minimum of 0.

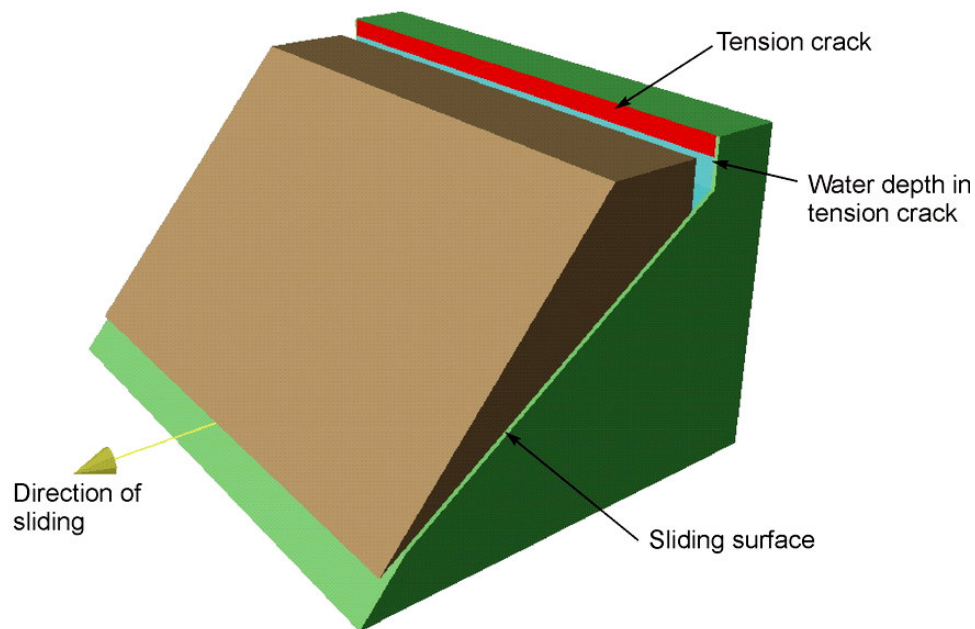


Figure 3: RocPlane model of Sau Mau Ping slope.

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Using the distributions shown in Figure 2, the RocPlane model shown in Figure 3 was used, with Latin Hypercube sampling, to carry out 5,000 iterations on the factor of safety. The resulting probability distribution is plotted in Figure 4. This histogram gives a mean factor of safety of 1.34 with a standard deviation of 0.23, a minimum of 0.61 and a maximum of 2.33. The best fit distribution is a beta distribution with the same mean, standard deviation, minimum and maximum.

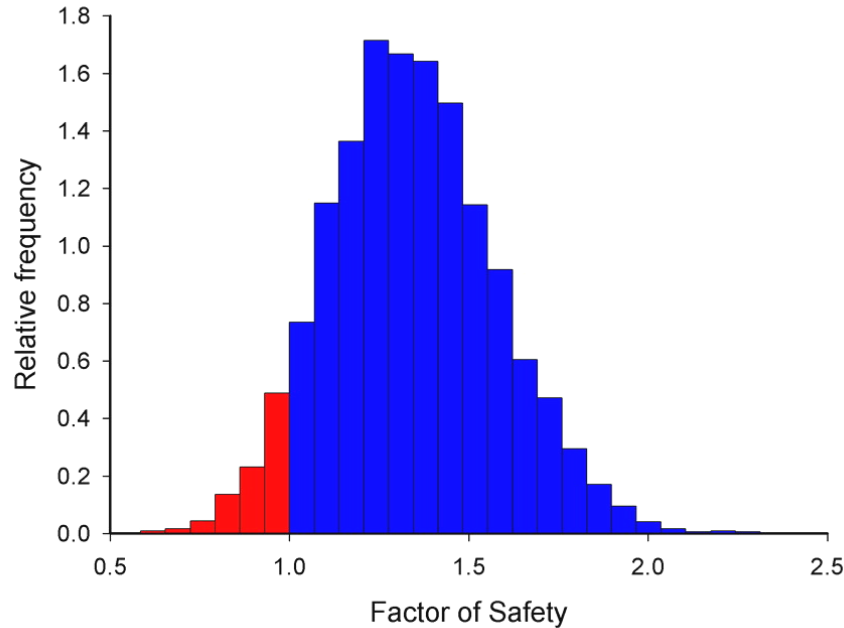


Figure 4: Distribution of the factor of safety for the Sau Mau Ping slope computed by means of the program RocPlane.

The calculated *probability of failure* is found to be 6.4% and is given by the ratio of the area under the distribution curve for  $F < 1$  (shown in red in Figure 4) divided by the total area under the distribution curve. This means that, for the combination of slope geometry, shear strength, water pressure and earthquake acceleration parameters assumed, 64 out of 1000 similar slopes could be expected to fail at some time during the life of the slope. Alternatively, a length of 64 m could be expected to fail in every 1000 m of slope.



This is a reasonable risk of failure for short term conditions and a risk of this magnitude may be acceptable in an open pit mine, with limited access of trained miners, and even on a rural road. However, in the long term, this probability of failure is not acceptable for a densely populated region such as Kowloon. As described in the chapter “A slope stability problem in Hong Kong”, remedial measures were taken to improve the long term stability of the slope and the effectiveness of these remedial measures could be evaluated using the same probabilistic techniques as described above.

### **Acknowledgements**

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## Dr. Evert Hoek: Experience and Expertise

Evert Hoek was born in Zimbabwe, graduated in mechanical engineering from the University of Cape Town and became involved in the young science of rock mechanics in 1958, when he started working in research on problems of brittle fracture associated with rockbursts in very deep mines in South Africa.

His degrees include a PhD from the University of Cape Town, a DSc (eng) from the University of London, and honorary doctorates from the Universities of Waterloo and Toronto in Canada. He has been elected as a Fellow of the Royal Academy of Engineering (UK), a Foreign Associate of the US National Academy of Engineering and a Fellow of the Canadian Academy of Engineering.

Dr. Hoek has published more than 100 papers and 3 books. He spent 9 years as a Reader and then Professor at the Imperial College of Science and Technology in London, 6 years as a Professor at the University of Toronto, 12 years as a Principal of Golder Associates in Vancouver, and the last 17 years as an independent consulting engineer based in North Vancouver. His consulting work has included major civil and mining projects in 35 countries around the world and has involved rock slopes, dam foundations, hydroelectric projects, underground caverns and tunnels excavated conventionally and by TBM.

Dr. Hoek has now retired from active consulting work but, in 2010, is still a member of consulting boards on three major civil and mining engineering projects in Canada, the USA and Chile.

